

Optimization of Multilayer Standby Mechanisms in Continuous Chemical Processes

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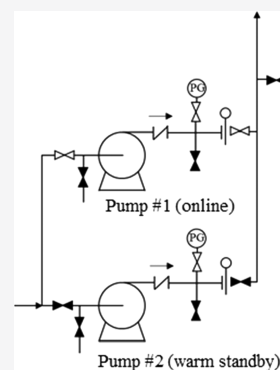
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ABSTRACT: Every critical online unit in a continuous process must always function normally, and one or more identical units are usually put on standby to sustain the uninterrupted operation. Although a few related studies have been reported in the literature, a comprehensive analysis of the standby mechanism still has not been carried out. The objective of this research is to construct a generalized mathematical model to synthesize the multilayer standby mechanisms for any given processes by minimizing the total expected life cycle expenditure. A Matlab code can be developed accordingly to perform the required optimization tasks via a genetic algorithm. The feasibility and effectiveness of the proposed approach have been demonstrated with the case studies concerning the pump system in a typical chemical plant. From the optimization results, one can obtain the optimal design specifications of the multilayer standby mechanism, which include (1) the number of layers, (2) the numbers of both online and spare sensors in each measurement channel, (3) the corresponding voting-gate logic in each channel, (4) the inspection interval of a switch, (5) the number of spares for a switch, (6) the inspection intervals for warm standbys, and (7) the number of cold standbys.



1. INTRODUCTION

According to Zhang et al.,¹ there are three basic types of standby mechanisms. “Hot standbys” refer to a collection of identical components arranged in parallel and all are loaded equally online. On the other hand, although the equipment in operation and its “warm standby” are both accessible in the continuous process, the latter is under a much lighter load and thus its failure rate is considerably lower. If the former component is broken, then the warm standby is supposed to take its place right away. Finally, the “cold standby” of an online component is treated in this work as an offline spare stored in the shop and its failure rate is very close to zero.

The critical units in a continuous chemical process are often protected with a variety of standbys. Since every such unit is required to function uninterruptedly, one or more standby may have to be incorporated into design for cases when the component under consideration is out of order during operation. Note that another class of protection mechanisms may also be present in the chemical plants for an entirely different purpose, i.e., emergency shutdown. Since the related issues have already been addressed thoroughly, e.g., see Liang and Chang,² Liao and Chang,³ Wibisono et al.,⁴ and Lepar et al.,⁵ this latter class is not discussed in the present paper.

Because the aforementioned standby mechanisms have not been analyzed rigorously in the past, the focus of this study is to develop a comprehensive mathematical programming model for assessing trade-offs in designs and maintenance policies. A full-fledged mechanism should include three distinct functions that are facilitated respectively by the sensing device(s), the switch, and the warm standby(s). Each of them may either fail safely (FS) or fail dangerously (FD), while only the latter are

considered in this study as the uncovered failure. An uncovered component fault may result in the entire system breakdown in spite of the presence of standbys. Such models are referred to as imperfect coverage (IPC) models.^{6–8} Depending on the type of fault tolerant techniques used, two major IPC models are usually adopted in the analysis of standby systems, i.e., element level coverage (ELC)⁹ and fault level coverage (FLC).^{10–12}

A common measure in industries is to introduce hardware redundancy (hot standbys) at the component level so as to achieve a desired availability at the system level. Multiple sensors and the corresponding voting gate¹³ may be installed to monitor the same process variable and to determine whether an unsafe condition is reached. On the basis of the assumption that the sensor failures are observable, their maintenance has often been carried out with the “corrective” repair strategy; i.e., the component is placed under repair as soon as its failure is revealed. Liang and Chang² proposed to incorporate spares (cold standby) in this practice for improving the overall reliability/availability of any given sensing device, while Liao and Chang³ extended the spare-supported corrective maintenance strategy to the multichannel monitoring systems.

Since the FD failures of switches and standby units can usually be regarded as unobservable or hidden, it is assumed in this study that they are all maintained with preventive policies. Specifically, every component should be scheduled to be

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inspected at predetermined time intervals. The failed unit must be repaired or replaced immediately after inspection, while the normal ones are allowed to stay online. Vaurio¹⁴ suggested that the inspection intervals must be determined to minimize the cost rate or accident rate, and incorporated the age replacement policy into the preventive maintenance scheme. Under this policy, every component is replaced after a fixed number of inspections and/or repairs, even if it is still functional. Badía et al.¹⁵ assumed that only the unrevealed failures may occur in the given system, and then developed a computational procedure to determine the cost-optimal inspection interval. Duarte et al.¹⁶ optimized the preventive maintenance strategy to achieve minimum total cost under the assumption that the repair rate is constant, and both failure rate and hazard rate increase over time.

To relieve the laborious manual effort, several mathematical programming models have been made available to automatically generate the optimal designs and maintenance policies for the safety interlocks. Liang and Chang² developed an integer programming model to optimize the structures and maintenance policies of multilayer protective systems, while Liao and Chang³ later amended this model so as to extend its applications to the multichannel ones. Wibisono et al.⁴ further considered components with time-dependent failure rates in this model by incorporating the Weibull distribution. Although successful applications of the above models have been reported extensively, they still cannot be directly applied to the standby systems. In fact, such a comprehensive model is not available at all and the related studies in the literature were concerned with only various special features of the standby mechanisms without the maintenance programs. Following is a brief literature summary:

Nakagawa and Osaki¹⁷ modeled two-component standby systems on as a Markov renewal process and derived the corresponding failure and repair rates. As a result, the time-dependent unavailability can be computed accordingly. Nakagawa¹⁸ suggested that switch malfunctions may be attributed to two different modes: (1) the permanent hardware failures and (2) the temporary operation errors. Pan¹⁹ studied the impacts of failure rates of the sensors, the imperfect switches, and the warm standbys on system availability. Raje et al.²⁰ performed the availability analysis of a two-component pump standby system on the basis of a Markov state-transition diagram. Yun and Cha²¹ analyzed the effects of switching time to the availability of a two-component warm standby system based on three different switch models. Levitin et al.²² incorporated random replacement times into the design of 1-out-of- N warm standby systems. Zhong and Jin²³ used semi-Markov theory to analyze cold standby availability using a preventive maintenance policy. Hellmich and Berg²⁴ carried out Markov analysis of redundant standby safety systems under periodic testing. Levitin et al.²⁵ considered the allocation problems of heterogeneous warm standby series-parallel systems. Zhu et al.²⁶ discussed the optimal design of redundant systems by incorporating various costs. Kayedpour et al.²⁷ presented a multiobjective optimization strategy for the standby systems, which maximized the total availability at the lowest possible cost by making use of the Markov theory and nondominated sorting genetic algorithm (NSGA-II). Reliability analyses of hierarchical system are also studied in a number of studies.^{28–30} The designs of standby mechanisms using the multivalued decision diagram (MDD) technique have also been reported extensively.^{9,11,12,31} However, most of them only discussed the reliability of a standby system in which the configuration is fixed. It is thus difficult to

use MDDs to optimize the configurations of multilayer standby mechanisms directly.

From the above discussions, it is clear that there are strong incentives to develop a comprehensive programming model for automatically synthesizing the optimal designs and maintenance policies of the standby mechanisms in continuous processes. For this purpose, the rest of the paper is organized as follows. Section 2 gives a conceptual description of the standby system structure. Section 3 presents the superstructure of a single protection layer. Section 4 depicts a generalized event tree enumerating all failure-induced scenarios in the multilayer standby mechanism. Section 5 presents the model formulation for characterizing a single layer which consists of online unit, monitoring subsystem, switch, and warm standby. Section 6 provides the unified governing equations for calculating the probabilities of all scenarios in the multilayer standby mechanism. Section 7 depicts the objective function in the proposed mathematical program, i.e., the total expected life cycle expenditure, which includes the total expected life cycle loss, the expected life cycle cost of monitoring subsystems, the switches, and the warm standbys. Extensive case studies are presented in section 8 to show the feasibility and effectiveness of the proposed method. Conclusions are then summarized in section 9.

2. MULTILAYER STANDBY MECHANISMS IN CONTINUOUSLY OPERATED PLANTS

A warm standby mechanism, which is designed to withstand a constant load continuously over a designated time horizon, is

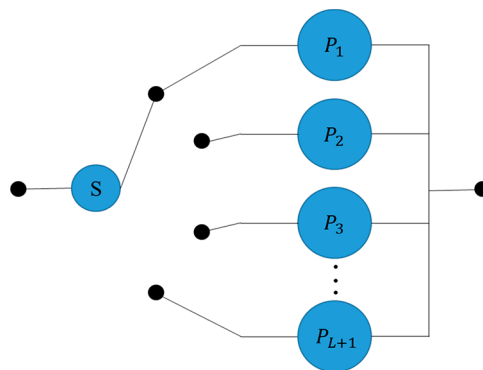


Figure 1. Conceptual mechanism of warm standby.

shown conceptually in Figure 1. In this representation, P_1 is the component in operation and its state is monitored with one or more sensors. The other identical components in this setup, i.e., P_2, P_3, \dots, P_{L+1} ($L \geq 1$), serve as the warm standbys, while S is a switch. In this study, L is considered to be the number of protection layers. If P_1 fails and the resulting sensor readings reveal an abnormal condition, then S should activate P_2 immediately. It is assumed that P_2 is also equipped with independent and dedicated sensor(s) and, when it fails at a later time, S is supposed to turn to the next component P_3 if available. It should be noted that, in some cases, the switching function can be performed manually by a human operator. In many chemical plants, a common example of the standby mechanism is the normally running pump together with its warm standby. Clearly such an arrangement is needed when the continuous flow of a critical process stream must be maintained at a constant level. As a second example, one may consider a jacketed continuous stirred tank reactor in which exothermic reaction (reactions)

takes (take) place. If the normal cooling supply stops unexpectedly, the water flow from an emergency head tank can be activated to temporarily keep the reactor in operation.

3. SUPERSTRUCTURE IN SINGLE PROTECTION LAYER

To illustrate the multilayer standby mechanism clearly, let us consider the superstructure of the standby mechanism in the l th layer in Figure 2.

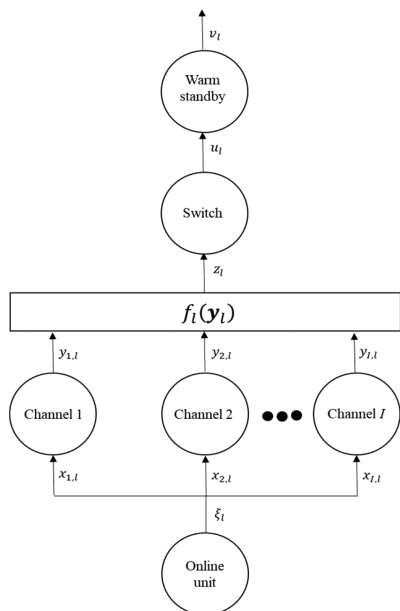


Figure 2. Superstructure of the standby mechanism in the l th layer.

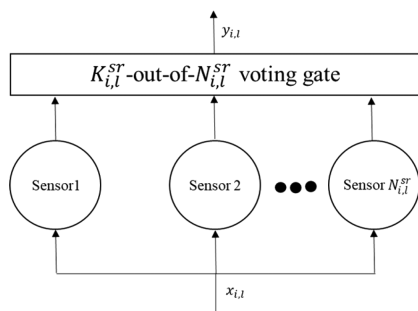


Figure 3. Superstructure of the i th measurement channel in the l th layer.

In this structure, ξ_l ($l = 1, 2, \dots, L$) is a binary variable which denotes whether the online unit of the l th layer is abnormal, i.e.

$$\xi_l = \begin{cases} 0, & \text{online unit of } l\text{th layer is normal} \\ 1, & \text{otherwise} \end{cases} \quad (1)$$

Another binary variable $x_{i,l}$ ($i = 1, 2, \dots, I$) is adopted to denote whether the i th variable of the l th layer stays within an acceptable range, i.e.

$$x_{i,l} = \begin{cases} 0, & \text{ith variable of } l\text{th layer stays within an acceptable range} \\ 1, & \text{otherwise} \end{cases} \quad (2)$$

It is assumed that each of the above variables is monitored with identical online sensors. This collection of hardware items as a whole is referred to in this paper as a “measurement

channel”, and Figure 3 shows the superstructure of a measurement channel. Since measurement errors are unavoidable, hardware redundancy is introduced to enhance reliability. Specifically, a total of $N_{i,l}^{sr}$ identical sensors are installed in the i th channel of the l th layer. Each channel is equipped with a designer-specified $K_{i,l}^{sr}$ -out-of- $N_{i,l}^{sr}$ voting gate to verify whether the variable in question stays within an acceptable range. A binary vector $y_l = \langle y_{1,l}, y_{2,l}, \dots, y_{I,l} \rangle$ is used to characterize all channel outputs, i.e.

$$y_{i,l} = \begin{cases} 0, & \text{ith channel of } l\text{th layer indicates safe state} \\ 1, & \text{otherwise} \end{cases} \quad (3)$$

Next, all channel outputs in the same layer are fed into the alarm function $f_l(y_l)$, i.e.

$$f_l(y_l) = \begin{cases} 0, & \text{alarm of } l\text{th layer is not activated} \\ 1, & \text{otherwise} \end{cases} \quad (4)$$

To simplify the model formulation, the “OR” logic is adopted in every such function, i.e., the function output z_l is expressed as

$$z_l = f_l(y_l) = \begin{cases} 0, & \forall y_{i,l} = 0 \\ 1, & \exists y_{i,l} = 1 \end{cases} \quad (5)$$

In other words, a switching action to turn on the warm standby in the l th layer should be carried out when $z_l = 1$. A binary variable u_l is used to denote whether the switching operation is successful, i.e.

$$u_l = \begin{cases} 0, & \text{switching operation of } l\text{th layer does not take place} \\ 1, & \text{otherwise} \end{cases} \quad (6)$$

Finally, an additional binary variable v_l is introduced to indicate whether the warm standby of the l th layer is activated online, i.e.

$$v_l = \begin{cases} 0, & \text{warm standby of } l\text{th layer remains idle} \\ 1, & \text{otherwise} \end{cases} \quad (7)$$

4. GENERALIZED EVENT TREE

Let us first denote the life cycle duration of the plant operation as H (years), where $0 < t \leq H$. In this proposed model, H should be regarded as a predetermined constant parameter which equals the operating horizon between two consecutive routine shutdowns of the given plant. Note that the life cycle duration should be a value estimated according to engineering knowledge or operation experience. Thus, there may be a slight difference between the actual and predicted life cycle durations.

To exhaustively enumerate all failure-induced scenarios in the standby mechanism over the time interval $[0, t]$, let us consider the generalized event tree in Figure 4. In Figure 4, τ_l is used to denote the switching time of the warm standby in the l th layer ($l = 1, 2, \dots, L$) and $0 < \tau_1 < \tau_2 < \dots < \tau_L < t$. It should be noted that the primary online unit in layer l is assumed to be not broken in the time interval $[\tau_{l-1} + \varepsilon_{l-1} + \varepsilon'_{l-1}, \tau_l)$. Note also that two small time intervals, i.e., $[\tau_l, \tau_l + \varepsilon_l)$ and $[\tau_l + \varepsilon_l, \tau_l + \varepsilon_l + \varepsilon'_l)$, are adopted in layer l to explicitly represent the precedence order of the initial failure of the primary unit and the subsequent responses of sensing, switching, and activating devices. Since these events take place almost instantaneously, it is clear that $\varepsilon_l \rightarrow 0$ and $\varepsilon'_l \rightarrow$

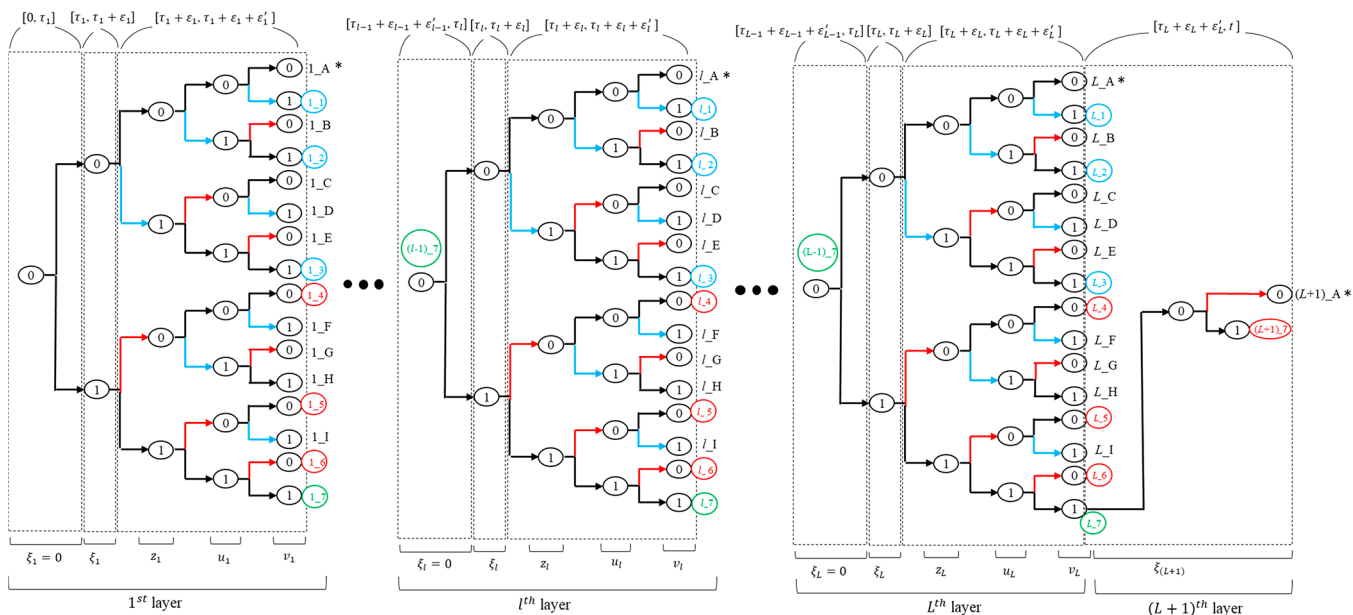


Figure 4. Generalized event tree for multilayer standby mechanism.

0. Finally, the interval $[\tau_L + \epsilon_L + \epsilon'_L, t]$ should be regarded as the time period in which all protection mechanisms are out of order.

Notice that the event-tree branches during the interval $[\tau_l + \epsilon_l, \tau_l + \epsilon_l + \epsilon'_l]$ in every layer are color coded. The black-colored branches denote that the corresponding instruments are functional, while the blue- and red-colored branches represent their FS and FD failures, respectively. Thus, the $L + 1$ asterisk-labeled branches in the tree can be thought of as the anticipated event sequences. The l th ($l = 1, 2, \dots, L + 1$) one among them describes the scenario that the primary online unit in layer l is still working at time τ_l and thus the warm standby remains idle afterward.

To simplify model formulation, it is assumed in this study that the probability of two or more instrument failures occurring within the small interval between $\tau_l + \epsilon_l$ and $\tau_l + \epsilon_l + \epsilon'_l$ is extremely low and, thus, the corresponding branches in the aforementioned event tree can be omitted. Specifically, the eight scenarios corresponding to branches l_B, l_C, \dots, l_H , and l_I ($l = 1, 2, \dots, L$) are ignored in this study. The other scenarios in each layer are labeled numerically. Branches l_1, l_2 , and l_3 ($l = 1, 2, \dots, L$) can be regarded as the FS failures of the overall standby mechanism, while branches l_4, l_5 , and l_6 ($l = 1, 2, \dots, L$) are treated as the corresponding FD failures. On the other hand, branch l_7 ($l = 1, 2, \dots, L$) is used to represent the scenario that warm standby is activated successfully in layer l after the primary online unit failed. This unit is then treated as the primary online unit in the next layer. Finally, if $l = L$, then the resulting online unit is unprotected in the time interval $[\tau_L + \epsilon_L + \epsilon'_L, t]$ and thus the unique scenario $(L + 1)_7$ should be analyzed individually.

5. MODEL FORMULATION FOR CHARACTERIZING A SINGLE PROTECTION LAYER

In this section, let us revisit the standby mechanism of the l th layer in Figures 2–4 and construct the model of each component in the single-layer superstructure.

5.1. Online Units. Let us further assume that online unit P_l ($l = 1, 2, \dots, L$) of each layer is functional when it is turned on at time τ_{l-1} , and assign T_l to be the age of P_l just before its failure. T_l is a random variable which may follow any given distribution,

and the probability that P_l fails during the time interval $(\tau_{l-1}, t]$ can be expressed as

$$\Phi_{T_l}(t) = \Pr\{T_l < t - \tau_{l-1}\} \tag{8}$$

where t (years) can be any instance after τ_{l-1} during operation, i.e., $0 < \tau_{l-1} < t \leq H$; $\Phi_{T_l}(t)$ denotes the cumulative distribution function for T_l . Let us next consider a particular instance τ_l (years) in the interval $(\tau_{l-1}, t]$. The probability of the event that online unit P_l fails within $(\tau_l, \tau_l + d\tau_l]$ should be

$$\Pr\{\xi_l(\tau_l) = 1\} = \left. \frac{d\Phi_{T_l}(t)}{dt} \right|_{t=\tau_l} d\tau_l = \varphi_{T_l}(\tau_l) d\tau_l \tag{9}$$

where

$$\varphi_{T_l}(\tau_l) = \left. \frac{d\Phi_{T_l}}{dt} \right|_{t=\tau_l}$$

is the probability density of failure at τ_l and $\xi_l \in \{0, 1\}$ is the binary variable denoting whether or not P_l fails at time τ_l .

5.2. Monitoring Subsystem. The FS conditional probability of monitoring subsystem can be expressed as

$$\begin{aligned} \text{PFS}_{\text{sr}}^{(l)}(\tau_l) &= \Pr\{f_l(\mathbf{y}_l(\tau_l)) = 1 | T_l > \tau_l\} \\ &= \Pr\{f_l(\mathbf{y}_l(\tau_l)) = 1 | \xi_l(\tau_l) = 0\} \end{aligned} \tag{10}$$

Under the assumption that each measurement channel works independently, the conditional FS probability of an OR-logic based monitoring system can be represented with the following formulas:

$$\text{PFS}_{\text{sr}}^{(l)}(\tau_l) = 1 - \prod_{i=1}^I (1 - A_{\text{AL},i}^{(l)}(\tau_l)) \tag{11}$$

where

$$A_{\text{AL},i}^{(l)}(\tau_l) = \Pr\{y_{i,l}(\tau_l) = 1 | \xi_l(\tau_l) = 0\} \tag{12}$$

Notice also that FS conditional probability of the i th measurement channel in the l th layer can be determined according to the general structure described in Figure 3. Specifically, if a $K_{i,l}$ -out-of- $N_{i,l}^{sr}$ voting gate is adopted in this channel, then

$$A_{AL,i}^{(l)}(\tau_l) = \sum_{j=K_{i,l}}^{N_{i,l}^{sr}} \frac{N_{i,l}^{sr}!}{j!(N_{i,l}^{sr}-j)!} [a_{sr,i}(\tau_l)]^j [1 - a_{sr,i}(\tau_l)]^{N_{i,l}^{sr}-j} \quad (13)$$

where $a_{sr,i}(\tau_l)$ denotes the FS probability of a single sensor in measurement channel i at time τ_l . For model simplicity, it is assumed in this study that the FS error of each sensor is temporary (i.e., it is due to spurious measurement signals only and the subsequent repair is not necessary) and its random occurrence time is uniformly distributed over a long period of time with a constant probability density $c_{AL,i}$. As a conservative estimate, let us set $a_{sr,i}(\tau_l) \approx a_{sr,i}(H) = c_{AL,i}H < 1$.

On the other hand, the FD conditional probability of this monitoring subsystem in the l th layer at time τ_l can be expressed as follows:

$$\begin{aligned} \text{PFD}_{sr}^{(l)}(\tau_l) &= \Pr\{f_l(\mathbf{y}_l(\tau_l)) = 0 | T_l = \tau_l\} \\ &= \Pr\{f_l(\mathbf{y}_l(\tau_l)) = 0 | \xi_l(\tau_l) = 1\} \end{aligned} \quad (14)$$

By assuming that the measurement channels yield independent outputs, the conditional FD probability of an OR-logic based monitoring system can be formulated as follows:

$$\text{PFD}_{sr}^{(l)} = \prod_{i=1}^I B_{AL,i}^{(l)} \quad (15)$$

where

$$B_{AL,i}^{(l)}(\tau_l) = \Pr\{y_{i,l}(\tau_l) = 0 | \xi_l(\tau_l) = 1\} \quad (16)$$

In the present study, the FD sensor failures are considered to be permanent and, thus, a spare-supported maintenance scheme is applied to every measurement channel. The *time-dependent* FD conditional probability of the i th measurement channel in the l th layer, i.e., $B_{AL,i}^{(l)}(\tau_l)$, can be computed according to the model formulation presented in part A-1 of the Supporting Information.

5.3. Switch. The mathematical expression of the FS conditional probability of switch in the l th layer at time τ_l can be constructed with a similar rationale, that is

$$\text{PFS}_{sw}^{(l)}(\tau_l) = \Pr\{u_l(\tau_l) = 1 | f_l(\mathbf{y}_l(\tau_l)) = 0\} \quad (17)$$

If we assume the switching function can be performed manually by an operator or automatically by a hardware component, $\text{PFS}_{sw}^{(l)}(\tau_l)$ can be written as

$$\text{PFS}_{sw}^{(l)}(\tau_l) = a_{sw}(\tau_l) \quad (18)$$

where $a_{sw}(\tau_l)$ denotes the single-switch FS probability. For simplicity, let us also assume that the occurrence time of FS switching error is uniformly distributed with a constant density c_{sw} . Therefore, $a_{sw}(\tau_l) \approx c_{sw}H$ is a conservative estimate. On the other hand, the corresponding FD conditional probability of switches in the l th layer at time τ_l can be written as

$$\text{PFD}_{sw}^{(l)}(\tau_l) = \Pr\{u_l(\tau_l) = 0 | f_l(\mathbf{y}_l(\tau_l)) = 1\} \quad (19)$$

This time-dependent conditional probability can be computed on the basis of a constant failure rate λ_{sw} (year⁻¹) and the mathematical model of a spare-supported preventive maintenance policy without repairs (see part A-2 of the Supporting Information).

5.4. Warm Standby. It is assumed that the warm standby may go online by itself without activating the switch, and such an incident is considered to be an FS error of the warm standby in the present study. On the other hand, a corresponding FD failure should be concerned with the scenario that the warm standby remains idle even after a switching action is carried out. Based on the above qualifications, the conditional FS and FD probabilities of warm standbys in the l th layer can be written respectively as follows:

$$\text{PFS}_{wb}^{(l)}(\tau_l) = \Pr\{v_l(\tau_l) = 1 | u_l(\tau_l) = 0\} \quad (20)$$

$$\text{PFD}_{wb}^{(l)}(\tau_l) = \Pr\{v_l(\tau_l) = 0 | u_l(\tau_l) = 1\} \quad (21)$$

Because there is only one warm standby in each layer, $\text{PFS}_{wb}^{(l)}$ should be equal to the FS probability of a single warm standby, i.e.

$$\text{PFS}_{wb}^{(l)}(\tau_l) = a_{wb}(\tau_l) \quad (22)$$

where $a_{wb}(\tau_l)$ denotes the FS probability of a single warm standby. Again for the sake of simplicity, it is assumed that the random occurrence time of FS operation error of the warm standby is uniformly distributed with a constant density c_{wb} . Therefore, $a_{wb}(\tau_l) \approx c_{wb}H$ is a conservative estimate. On the other hand, the time-dependent FD conditional probability in eq 22 can be determined on the basis of a constant failure rate λ_{wb} (year⁻¹) and the mathematical model of a spare-supported preventive maintenance strategy with repairs (see part A-3 of the Supporting Information).

6. MATHEMATICAL DESCRIPTION OF FAULT PROPAGATION SCENARIOS

In this section, our objective is to determine the probabilities of all fault propagation scenarios that are not negligible, i.e., branches l_1-l_6 ($l = 1, 2, \dots, L$) and $(L+1)_7$ in Figure 4. For convenience of illustration, let us assume that T_l is exponentially distributed,³² i.e.

$$\Phi_{T_l}(t) = \Pr\{T_l < t - \tau_{l-1}\} = 1 - e^{-\lambda_l(t - \tau_{l-1})} \quad (23)$$

$$\Pr\{\xi_l(\tau_l) = 1\} = \frac{d\Phi_{T_l}(t)}{dt} \Big|_{t=\tau_l} d\tau_l = \lambda_l e^{-\lambda_l(\tau_l - \tau_{l-1})} d\tau_l \quad (24)$$

where λ_l (year⁻¹) is the constant failure rate of P_l if the random variable T_l follows an exponential distribution. For the sake of conciseness, listed below are only the generalized governing equations for computing the probabilities of all fault propagation scenarios. Using the three-layer standby mechanism ($L = 3$) as an illustrative example, the detailed derivation can be found in part A-4 of the Supporting Information.

- Repetitive scenarios ($l = 1, 2, \dots, L$):

Scenario l_1 : $\xi_l(\tau_l) = 0$, $z_l(\tau_l) = 0$, $u_l(\tau_l) = 0$, $v_l(\tau_l) = 1$.

$$\begin{aligned} \frac{dP_{l-1}^{LS}(t)}{dt} &= [e^{-\lambda_l t}] [1 - \text{PFS}_{sr}^{(l)}(t)] [1 - \text{PFS}_{sw}^{(l)}(t)] [c_{wb}^{(l)}] P_{l-1}^*(t) \end{aligned} \quad (25)$$

Scenario l_2 : $\xi_i(\tau_i) = 0, z_i(\tau_i) = 0, u_i(\tau_i) = 1, v_i(\tau_i) = 1$.

$$\frac{dP_{l_2}^{LS}(t)}{dt} = [e^{-\lambda t}][1 - PFS_{sr}^{(l)}(t)][c_{sw}^{(l)}][1 - PFD_{wb}^{(l)}(t)]P_{l-1}^*(t) \quad (26)$$

Scenario l_3 : $\xi_i(\tau_i) = 0, z_i(\tau_i) = 1, u_i(\tau_i) = 1, v_i(\tau_i) = 1$.

$$\frac{dP_{l_3}^{LS}(t)}{dt} = [e^{-\lambda t}][c_{sr}^{(l)}][1 - PFD_{sw}^{(l)}(t)][1 - PFD_{wb}^{(l)}(t)]P_{l-1}^*(t) \quad (27)$$

Scenario l_4 : $\xi_i(\tau_i) = 1, z_i(\tau_i) = 0, u_i(\tau_i) = 0, v_i(\tau_i) = 0$.

$$\frac{dP_{l_4}^{LS}(t)}{dt} = [\lambda e^{-\lambda t}][PFD_{sr}^{(l)}(t)][1 - PFS_{sw}^{(l)}(t)][1 - PFS_{wb}^{(l)}(t)]P_{l-1}^*(t) \quad (28)$$

Scenario l_5 : $\xi_i(\tau_i) = 1, z_i(\tau_i) = 1, u_i(\tau_i) = 0, v_i(\tau_i) = 0$.

$$\frac{dP_{l_5}^{LS}(t)}{dt} = [\lambda e^{-\lambda t}][1 - PFD_{sr}^{(l)}(t)][PFD_{sw}^{(l)}(t)][1 - PFS_{wb}^{(l)}(t)]P_{l-1}^*(t) \quad (29)$$

Scenario l_6 : $\xi_i(\tau_i) = 1, z_i(\tau_i) = 1, u_i(\tau_i) = 1, v_i(\tau_i) = 0$.

$$\frac{dP_{l_6}^{LS}(t)}{dt} = [\lambda e^{-\lambda t}][1 - PFD_{sr}^{(l)}(t)][1 - PFD_{sw}^{(l)}(t)][PFD_{wb}^{(l)}(t)]P_{l-1}^*(t) \quad (30)$$

• Connective terms ($l = 1, 2, \dots, L - 1$):

$$\frac{dP_l^*(t)}{dt} = [\lambda e^{-\lambda t}][1 - PFD_{sr}^{(l)}(t)][1 - PFD_{sw}^{(l)}(t)] [1 - PFD_{wb}^{(l)}(t)][e^{\lambda(t+1)^t}]P_l^{**} \quad (31)$$

$$\frac{dP_l^{**}(t)}{dt} = [\lambda_{(l-1)}e^{-\lambda_{(l-1)}t}][1 - PFD_{sr}^{(l-1)}(t)] [1 - PFD_{sw}^{(l-1)}(t)][1 - PFD_{wb}^{(l-1)}(t)]P_{l-1}^{**} \quad (32)$$

Note that it is required that $P_0^*(t) = 1$ and $P_1^{**}(t) = 1$ when $l = 1$.

• Unprotected layer:

$$\frac{dP_{(L+1)_7}^{LS}(t)}{dt} = \lambda_{(L+1)}P_{(L+1)_7}^{LS}(t) \quad (33)$$

$$\frac{dP_{(L+1)_7}^{LS}(t)}{dt} = [-\lambda_{(L+1)}P_{(L+1)_7}^{LS}(t)] + [\lambda_L e^{-\lambda_L t}][1 - PFD_{sr}^{(L)}(t)][1 - PFD_{sw}^{(L)}(t)] [1 - PFD_{wb}^{(L)}(t)]P_{(L-1)}^*(t) \quad (34)$$

7. OBJECTIVE FUNCTION

For the design of any normal unit in chemical processes, e.g., a heat exchanger, a distillation column, or a reactor, it is usually assumed that this unit is either too expensive to install a standby system or very reliable so that the chance of its failure is extremely low. As a result, the normal design often deals with minimization of total annual cost (TAC) which includes the

annualized capital cost and the annual operating cost. On the other hand, the objective function of the standby design is the total expected life cycle expenditure, which includes the total expected life cycle loss (IS^H), the expected life cycle cost of the monitoring subsystem (LCC^{sr}), the expected life cycle cost of switches (LCC^{sw}), and the expected life cycle cost of the standby subsystem (LCC^{wb}), i.e.

$$obj = IS^H + LCC^{sr} + LCC^{sw} + LCC^{wb} \quad (35)$$

If the initial budget (PC_{ibc}) is limited, we need to impose an additional constraint as follows

$$PCT^{sr} + PCT^{sw} + PCT^{wb} \leq PC_{ibc} \quad (36)$$

where PCT^{sr} , PCT^{sw} , and PCT^{wb} are the total purchase costs of the monitoring subsystem, switches, and standby subsystem, respectively. The consideration of expected loss in each fault propagation scenario is the major difference in objective function between the optimization problems of normal and standby designs. Notice also that the constraints of standby design involve both the random and deterministic variables, while the constraints of normal design only involve the deterministic variables.

7.1. Total Expected Life Cycle Loss. After obtaining all scenario probabilities by solving the ordinary differential equations in section 6 and parts A-1 to A-3 in Supporting Information simultaneously, one can determine the total expected loss in the k th year as follows:

$$IS(k-1, k) = C_a^{(k)} \int_{k-1}^k \left[\sum_{l=1}^L (P_{l-1}^{LS} + P_{l-2}^{LS} + P_{l-3}^{LS}) \right] dt + C_b^{(k)} \int_{k-1}^k \left[P_{(L+1)_7}^{LS} + \sum_{l=1}^L (P_{l-4}^{LS} + P_{l-5}^{LS} + P_{l-6}^{LS}) \right] dt \quad (37)$$

where $C_a^{(k)}$ and $C_b^{(k)}$ are the FS and FD losses of the multilayer standby in the k th year, respectively. If the life cycle is H years, the total expected life cycle loss can be expressed as

$$IS^H = \sum_{k=1}^H IS(k-1, k)CF_k \quad (38)$$

where CF_k is the yearly factor to convert the expenditure in the k th year to the present value.

7.2. Expected Life Cycle Cost of Monitoring Subsystem. The expected life cycle cost of a monitoring subsystem can be further classified into three types, i.e., the purchase cost, the expected repair cost, and the expected replacement cost. Let us assume that there are $N_{i,l}^{sr}$ online sensors and $SN_{i,l}^{sr}$ spare sensors in the i th channel of the l th layer. If the purchase cost of a sensor in the i th channel is PC_i^{sr} , the total purchase cost of the monitoring subsystem can be expressed as follows:

$$PCT^{sr} = \sum_{l=1}^L \sum_{i=1}^I (N_{i,l}^{sr} + SN_{i,l}^{sr})PC_i^{sr} \quad (39)$$

Let us denote the expected numbers of repairs and replacements in the i th channel of the l th layer in the k th year as $ENRr_{i,k,l}^{sr}$ and $ENRpl_{i,k,l}^{sr}$, respectively, and denote the corresponding repair and replacement costs of a sensor in the i th channel as RrC_i^{sr} and $RplC_i^{sr}$, respectively. The total expected repair and replacement costs of the monitoring subsystem can then be expressed respectively in eqs 40 and 41.

$$\text{RrCT}^{\text{sr}} = \sum_{l=1}^L \sum_{i=1}^I \sum_{k=1}^H \text{ENRr}_{i,k,l}^{\text{sr}} \text{RrC}_i^{\text{sr}} \text{CF}_k \quad (40)$$

$$\text{RplCT}^{\text{sr}} = \sum_{l=1}^L \sum_{i=1}^I \sum_{k=1}^H \text{ENRpl}_{i,k,l}^{\text{sr}} \text{RplC}_i^{\text{sr}} \text{CF}_k \quad (41)$$

Note that $\text{ENRr}_{i,k,l}^{\text{sr}}$ and $\text{ENRpl}_{i,k,l}^{\text{sr}}$ can be calculated according to the corresponding Markov's diagram if the repair rate, replacement rate, and failure rate of the specific sensor are given (see part A-1 of the Supporting Information). Consequently, the total expected life cycle cost of monitoring subsystem can be expressed as

$$\text{LCC}^{\text{sr}} = \text{PCT}^{\text{sr}} + \text{RrCT}^{\text{sr}} + \text{RplCT}^{\text{sr}} \quad (42)$$

7.3. Expected Life Cycle Cost of Switches. The expected life cycle cost of switches can be classified into the expected purchase cost and the inspection cost. It is assumed that the purchase cost of a switch is low enough and, thus, if an online switch is found to be defective, a new one can be adopted to replace it directly without repair. Let us also assume that, other than the online switch, there are SN^{sw} spares. Note that the expected number of spares (SN^{sw}) and the number of inspections (st^{sw}) can be calculated according to part A-2 of the Supporting Information. If the purchase cost of a switch is PC^{sw} , the total expected purchase cost of switches can be written as

$$\text{PCT}^{\text{sw}} = (1 + \text{SN}^{\text{sw}}) \text{PC}^{\text{sw}} \quad (43)$$

If the inspection cost of a switch is InspC^{sw} , the total inspection cost of the switches can be determined according to the following formula:

$$\text{InspCT}^{\text{sw}} = \sum_{k=1}^H \text{st}^{\text{sw}} \text{InspC}^{\text{sw}} \text{CF}_k \quad (44)$$

Therefore, the expected life cycle cost of switches can be expressed as follow:

$$\text{LCC}^{\text{sw}} = \text{PCT}^{\text{sw}} + \text{InspCT}^{\text{sw}} \quad (45)$$

7.4. Expected Life Cycle Cost of Standby Subsystem.

The expected life cycle cost of a standby subsystem can be divided into three parts, i.e. the purchase cost, the expected repair cost, and the inspection cost. Let us also assume that, other than the warm standby, there are SN_l^{wb} cold standbys in the l th layer. If the purchase cost of a standby unit is PC^{wb} , the total purchase cost of the standby subsystem can be expressed as follows:

$$\text{PCT}^{\text{wb}} = \sum_{l=1}^L (1 + \text{SN}_l^{\text{wb}}) \text{PC}^{\text{wb}} \quad (46)$$

Note also that the expected number of repairs of warm standby in the l th layer during the k th year ($\text{ENRr}_{k,l}^{\text{wb}}$) and the corresponding number of inspections of warm standby (st_l^{wb}) can both be computed according to part A-3 of the Supporting Information. Let us denote the repair and inspection costs of a standby unit as RrC^{wb} and InspC^{wb} respectively. The total expected repair and inspection costs of standby subsystem can then be expressed with eqs 47 and 48, respectively.

$$\text{RrCT}^{\text{wb}} = \sum_{l=1}^L \sum_{k=1}^H \text{ENRr}_{k,l}^{\text{wb}} \text{RrC}^{\text{wb}} \text{CF}_k \quad (47)$$

$$\text{InspCT}^{\text{wb}} = \sum_{l=1}^L \sum_{k=1}^H \text{st}_l^{\text{wb}} \text{InspC}^{\text{wb}} \text{CF}_k \quad (48)$$

Therefore, the expected life cycle cost of standby subsystem is

$$\text{LCC}^{\text{wb}} = \text{PCT}^{\text{wb}} + \text{RrCT}^{\text{wb}} + \text{InspCT}^{\text{wb}} \quad (49)$$

8. CASE STUDIES

The feasibility and effectiveness of the proposed design strategy are demonstrated with the case studies in this section. Figure 5

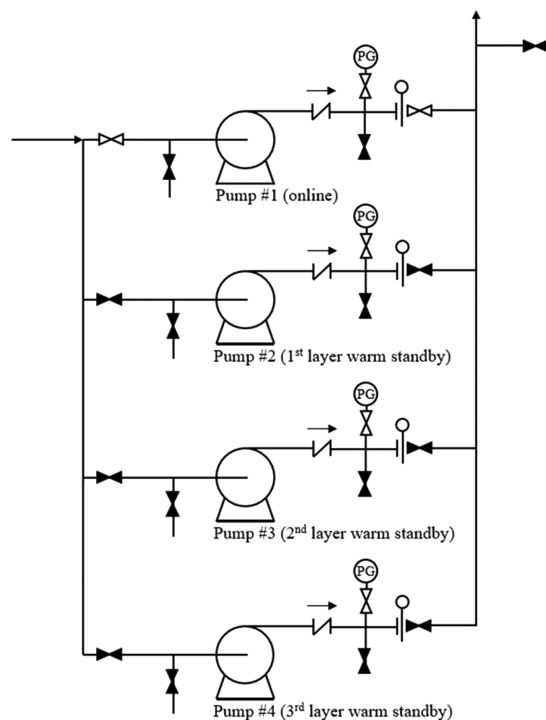


Figure 5. Pump system.

Table 1. Specifications of Sensors

	flow rate	motor speed
FD failure rate (year^{-1})	2.4	1.3
repair rate (year^{-1})	3	2.7
replacement rate (year^{-1})	365	365
purchase cost (USD)	90	250
repair cost (USD)	15	20
replacement cost (USD)	5	10
FS failure probability of a single sensor	0.3	0.1

Table 2. Specifications of Switches

FD failure rate (year^{-1})	1.5
purchase cost (USD)	100
inspection cost (USD)	10
FS failure probability of a single switch	0.4

shows a typical pump system in chemical plants. Pump 1 is running initially, and the others are the warm standbys. After executing a comprehensive hazard assessment, C_a is chosen to be 10^4 USD, and, for the purpose of comparison, two different fail-dangerous losses are chosen, i.e., $C_b = 10^6$ USD or $C_b = 10^8$ USD. Note that these financial losses are assumed to be constants over the entire operation horizon.

Table 3. Specifications of Warm Standbys

FD failure rate (year ⁻¹)	0.1
repair rate (year ⁻¹)	2.5
purchase cost (USD)	2,500
repair cost (USD)	100
inspection cost (USD)	50
FS failure probability of a single warm standby	0.2

Table 4. Optimization Results for Standby Mechanism Corresponding to $C_b = 10^6$ USD

	run no.			
	A-1	B-1	C-1	D-1
initial budget (USD)	none	12,000	7,000	6,000
total expected life cycle expenditure (USD)	44,668	45,058	71,512	84,135
purchase cost (USD)	12,190	11,940	6,420	5,980
maintenance cost (USD)	1,973	1,967	1,498	1,365
total expected life cycle loss (USD)	30,504	31,151	63,594	76,790
layer	2	2	1	1

Let us assume that there are two channels in the monitoring subsystems for measuring the motor speed and mass flow rate. The specifications of sensors, switches, and warm standbys are listed in Tables 1, 2, and 3, respectively. Note that the failure rate of an online unit is set to be twice as large as that of each warm standby (i.e., 0.2 year⁻¹).

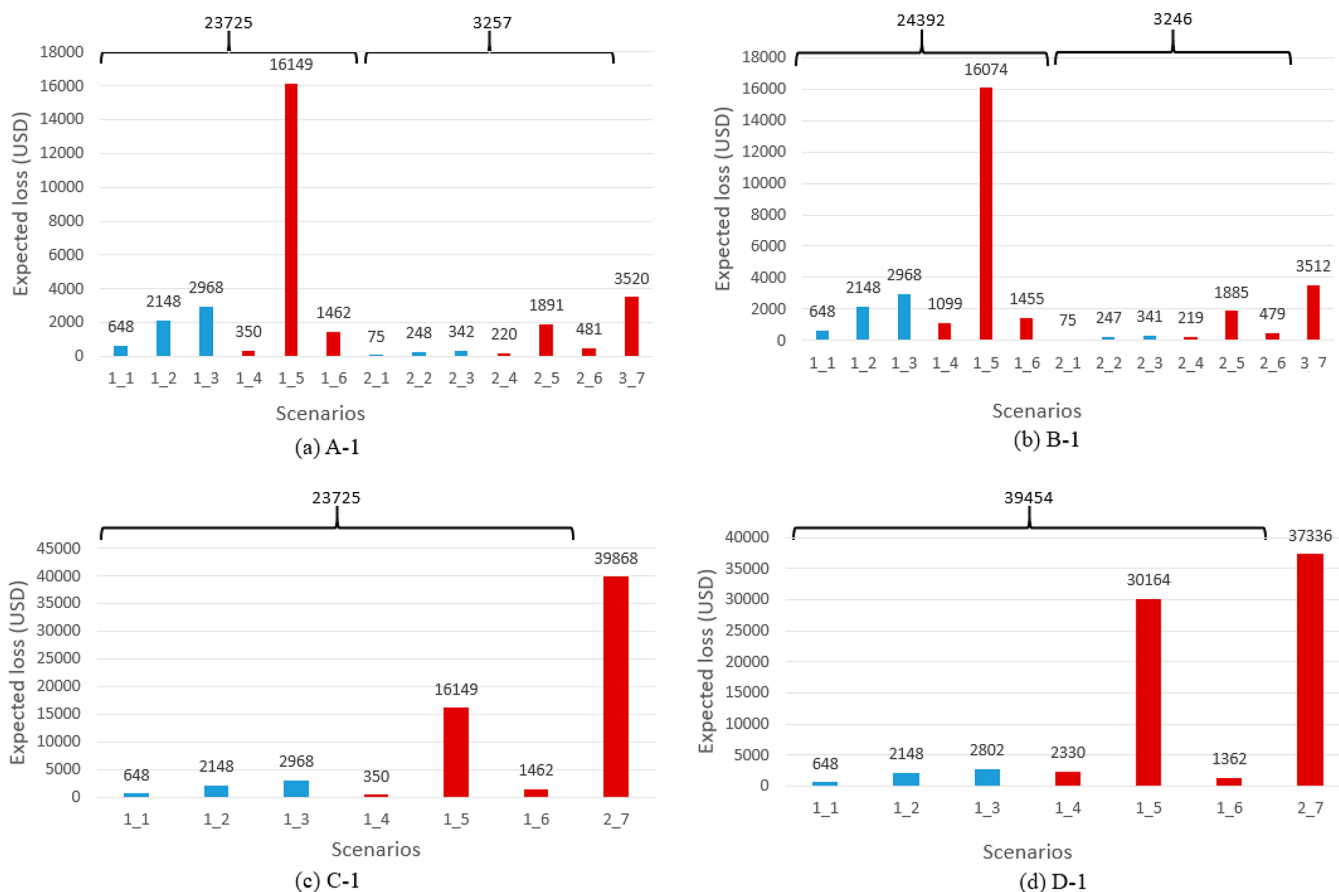
It is further assumed that the entire operational horizon (life cycle) is 2 years, i.e., $H = 2$, while the interest rate is 3%. The

Table 5. Optimization Results for Standby Mechanism Corresponding to $C_b = 10^8$ USD

	run no.			
	A-2	B-2	C-2	D-2
initial budget (USD)	none	12,000	7,000	6,000
total expected life cycle expenditure (USD)	2,084,184	2,466,633	5,796,605	7,132,101
purchase cost (USD)	18,460	11,940	6,420	5,980
maintenance cost (USD)	4,041	2,756	1,498	1,365
total expected life cycle loss (USD)	2,061,683	2,451,937	5,788,687	7,124,755
layer	3	2	1	1

maximum numbers of online sensors and spare sensors are both set to be 2 in each channel. It is also assumed that each warm standby in every layer is equipped with one cold standby. Furthermore, the maximum lengths of inspection intervals for switch and warm standbys are both set to be 4 months.

The numerical optimization runs were carried out with the genetic algorithm (GA)³³ in Matlab R2018b environment on an Intel Core i7 3.60 GHz PC. This Matlab code can also be developed for any other applications according to the generalized model presented in this paper and a user-specified maximum allowable layer number. In the case studies presented below, the maximum allowable number of protection layers is assumed to be three, i.e., $L \leq 3$. The corresponding Matlab code can be found in the Supporting Information. The parameter settings and convergence behaviors of the GA in case studies are

Figure 6. Expected losses of different scenarios ($C_b = 10^6$ USD).

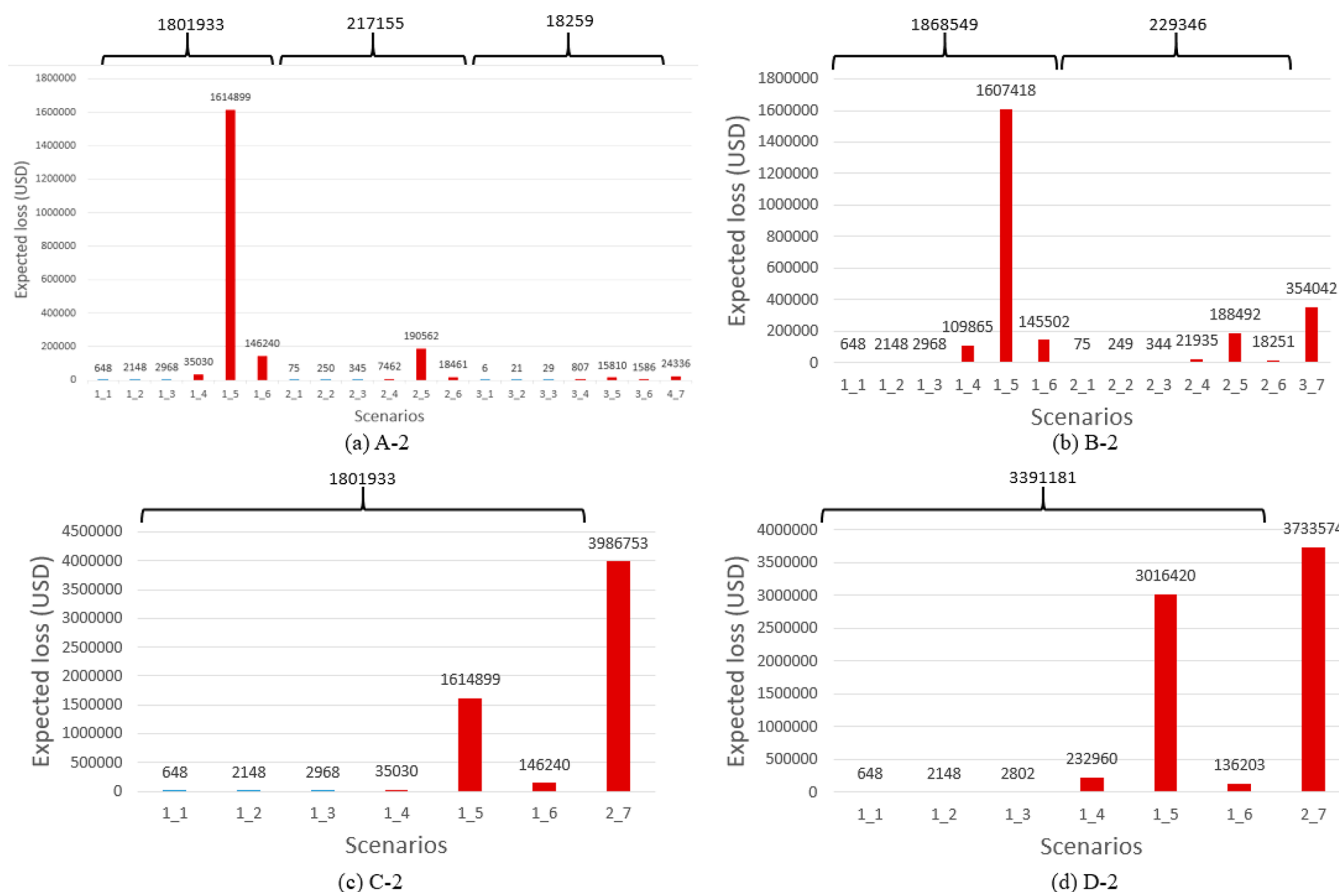


Figure 7. Expected losses of different scenarios ($C_b = 10^8$ USD).

described in part A-5 of the Supporting Information. The optimization results are summarized respectively in Table 4 for the case of $C_b = 10^6$ USD and Table 5 for the case of $C_b = 10^8$ USD. The corresponding optimal configurations are shown in part A-6 of the Supporting Information. Finally, Figures 6 and 7 show the expected losses of each and every scenario in the optimum solutions obtained with $C_b = 10^6$ USD and $C_b = 10^8$ USD, respectively. Also, these expected losses are marked in blue and red colors to distinguish the FS and FD failures, respectively. For the purpose of comparison, the sum of the expected losses in every layer is also presented at the tops of Figures 6 and 7.

From the aforementioned optimization results, several interesting observations can be made:

(a) From Figures 6 and 7, one can see that the FD loss of the multilayer standby is the major contributor of the total expected life cycle loss. Specifically, expected losses in scenarios l_5 ($l = 1, 2, \dots, L$) and $(L + 1)_7$ are considerably higher than those of other scenarios. The former scenarios can be attributed to the FD failures of switch, and their high expected losses are due to the relatively large failure rate (1.5 year^{-1}). It can also be noted from Tables A1 and A2 in the Supporting Information that the optimum inspection intervals for switch are at the preset lower bound ($T^{\text{sw}} = 1$ month) in most runs. These results imply that further shortening the inspection interval (if possible) may reduce the unavailability of the online switch and, thus, the expected losses in scenario l_5 ($l = 1, 2, \dots, L$). In scenario $(L + 1)_7$, which is associated with the outermost unprotected layer, since there is no standby pump available to ensure continuous operation, the corresponding expected loss can generally be lowered by installing an extra protection layer.

(b) From Tables 4 and 5, one can see that the objective value (i.e., total expected life cycle expenditure) of the standby mechanism can be reduced by relaxing the budget constraint, but this value eventually approaches a constant level after the budget exceeds a threshold. More specifically, the number of layers increases with the initial budget, but eventually reaches an upper bound. From the optimization results obtained with $C_b = 10^6$ USD and $C_b = 10^8$ USD, one can observe from Tables 4 and 5 that the optimum numbers of layers can be found to be two ($L = 2$) and three ($L = 3$) respectively when no initial budget limits are imposed. In the former case ($C_b = 10^6$), if one insists on using a larger number of layers when there is no budget limit, the objective value should be raised to a higher level. Specifically, an optimum three-layer standby mechanism ($L = 3$) requires a total expected life cycle expenditure of 47,729 USD (in which the purchase cost is 17,870 USD, the maintenance cost is 2,342 USD, and the total expected life cycle loss is 27,517 USD), and this value is larger than 44,668 USD in run A-1. On the other hand, notice that the latter case ($C_b = 10^8$) implies a more hazardous process and therefore more protection layers are called for to counter the detrimental outcomes.

(c) From Tables A1 and A2 in the Supporting Information, one should note that the purchase cost increases with the initial budget because increasing capital investment usually improves system reliability via a higher degree of hardware redundancy. On the other hand, if the initial budget is tightened, one should try to first cut down the spares to save the initial hardware cost.

(d) The total expected life cycle loss generally decreases with the increase of the initial budget. In addition, the higher the layer number in the standby mechanism, the lower the total expected

life cycle loss is. This is due to the fact that the expected losses of the scenarios corresponding to the outermost layer in Figure 4 can be further reduced by installing an extra layer. For example, in Figure 7, the expected loss of scenario 2_7 in run C-2 (i.e., 3,986,753 USD) can be reduced to 583,388 USD (total expected loss of the second layer in run B-2) by increasing the protection layer from $L = 1$ to $L = 2$. Similarly, the expected loss of scenario 3_7 in run B-2 (i.e., 354,042 USD) can be lowered to 42,595 USD (total expected loss of the third layer in run A-2) by increasing the protection layer from $L = 2$ to $L = 3$. However, one should note that there is a trade-off between the decrease of total expected life cycle loss and the increase of purchase cost.

(e) The standby structures are not sensitive to small life cycle variations. Specifically, the optimum designs without initial budget constraints remain unchanged when the life cycle duration varies between 1.9 and 2.1 years.

9. CONCLUSIONS

A generalized mathematical programming model and the corresponding Matlab code have been developed in this research to generate the optimum designs of multilayer standby mechanisms in continuous chemical processes systematically. One can apply this code in a wide range of applications without ad hoc approach. The feasibility and effectiveness of the proposed approach are demonstrated with case studies, i.e., the pump system in a typical chemical plant. From optimization results, one can determine the optimal number of protection layers, the corresponding hardware specifications, and the required maintenance policies automatically.

It should be also noted that the proposed standby mechanism is only suitable to withstand a constant load continuously over a designated time horizon. There are other equally realistic scenarios which require a different type of standby mechanism. In particular, this standby system should protect a continuous process against time-varying multilevel loads over a given horizon. The related issues are addressed in a future paper.

■ ASSOCIATED CONTENT

SI Supporting Information

The Supporting Information is available free of charge at <https://pubs.acs.org/doi/10.1021/acs.iecr.0c00233>.

Appendices and Matlab code (ZIP)

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Notes

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