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Game-theory based optimization strategies for stepwise development of indirect interplant heat integration plans

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ABSTRACT

Since the conventional design strategies for interplant heat integration usually focused upon minimization of the overall utility cost, the optimal solutions may not be implementable due to the additional need to distribute the financial benefits “fairly.” To resolve this profit sharing issue, a Nash-equilibrium constrained optimization strategy has already been developed to sequentially synthesize heat exchanger networks (HENs) that facilitate direct heat transfers across plant boundaries. Although this available approach is thermodynamically viable, the resulting network may be highly coupled and therefore inoperable. To address the operability issues in any multi-plant HEN, the present study aims to modify the aforementioned strategy by considering only *indirect* interplant heat-exchange options. Two separate sets of mathematical programming models are developed in this work for generating the total-site heat integration schemes with the available utilities and an extra intermediate fluid, respectively. The negotiation powers of the participating plants are also considered for reasonably distributing the utility cost savings and also shouldering the capital cost hikes. Finally, extensive case studies are presented to demonstrate the effectiveness of the proposed procedures and to compare the pros and cons of these two indirect heat-exchange alternatives.

1. Introduction

The total operating cost of almost every chemical plant can be largely attributed to the needs for heating and cooling utilities. The heat exchanger network (HEN) embedded in a chemical process is usually configured for the purpose of minimizing the utility consumption rates. A HEN design is traditionally produced with either a simultaneous optimization strategy [1] or a stepwise procedure for determining the minimum utility consumption rates and the minimum match number first [2] and then the network structure [3]. The former usually yields a better trade-off between utility and capital costs, but the computational effort required for solving the corresponding mixed-integer nonlinear programming (MINLP) model can be overwhelming. On the other hand, although only suboptimal solutions can be obtained in the latter case, implementing a stepwise method is much easier. For this very reason, a sequential approach is often adopted to configure the inner-plant heat-exchange networks in three steps. In the first two steps, a linear program (LP) and a mixed-integer linear program (MILP) are solved respectively to determine the minimum total utility cost and to identify the minimum number of matches and their heat duties [2]. A nonlinear programming (NLP) model is then solved in the final step for synthesizing the cost-optimal network [3].

Driven by the belief that significant extra benefit can be reaped by expanding the feasible region of any optimization problem, a number of studies have been carried out to develop various interplant heat integration schemes, e.g., see Bagajewicz and Rodera [4] and Anita [5] and Liew et al. [6]. The available synthesis methods for total site heat integration (TSHI) can be classified into three kinds: the insight-based pinch analysis [7], the model-based methods [8] and the hybrid methods [6], while the required interplant energy flows may be either realized with direct heat exchanges between process streams or facilitated indirectly with the extraneous fluids [9].

The main advantages of insight-based pinch analysis can be attributed to its target setting strategy and flexible design steps. Matsuda et al. [10] applied the area-wide pinch technology which incorporated the R-curve analysis and site-source-sink-profile analysis to TSHI of Kashima industrial area. For the fluctuating

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renewable energy supply, Liew et al. [11] proposed the graphical targeting procedures based on the time slices to handle the energy supply/demand variability in TSHI. In addition, a retrofit framework was proposed by the same research group [12] and the framework showed that energy retrofit projects should be approached from the total-site context first. Furthermore, Tarighaleslami et al. [13] developed a new improved TSHI method in order to address the non-isothermal utilities targeting issues.

On the other hand, the model-based methods are more rigorous and thus better equipped to identify the true optimum. Zhang et al. [14] proposed to use a superstructure for building a MINLP model to synthesize multi-plant HEN designs. Chang et al. [8] presented a simultaneous optimization methodology for interplant heat integration using the intermediate fluid circle(s). Wang et al. [9] adopted a hybrid approach for the same problems. The performances for heat integration across plant boundaries using direct, indirect and combined methods were analyzed and compared through composite curves, while the mathematical programming models were adopted to determine the optimal conditions of direct and/or indirect options [9].

As indicated in Cheng et al. [15], the aforementioned interplant heat integration arrangements were often not implementable in practice due to the fact that the profit margin might be unacceptable to one or more participating party. This drawback can be primarily attributed to the conventional HEN design objective, i.e., minimization of overall energy cost. Thus, the key to a successful interplant heat integration scheme should be to allow every plant to maximize its own benefit while striving for the largest overall saving at the same time. To address this benefit distribution issue, a game-theory based sequential optimization strategy has been developed by Cheng et al. [15] to generate the “fair” interplant integration schemes via *direct* heat exchanges between the hot and cold process streams across plant boundaries. In addition to a lighter computation load, this approach is justified by the fact that the game theoretic models can be more naturally incorporated into a step-by-step design practice when the same type of decision variables are evaluated one-at-a-time on a consistent basis. To be specific, let us consider their modeling strategy in more detail. After determining the global minimum of total utility cost with the LP model used in the first step of the conventional approach, a NLP model was then constructed for identifying the acceptable interplant heat flows in the given system. Since the commodities to be traded were energies of different grades, this model was formulated as a nonzero-sum matrix game, in which each game strategy was the fraction of heat flow entering/leaving a distinct temperature interval. On the basis of this conceptual analogy, the Nash equilibrium constraints [16] were imposed in the NLP model for solving the game while keeping the overall utility cost at minimum. It should be noted that, although Hiete et al. [17] also treated the benefit-sharing plan for interplant heat integration as a cooperative game, this alternative approach is less rigorous due to the requirements of heuristic manipulations. Finally, note that the game theory has been adopted in various other interplant resource integration applications, e.g., water network designs [18], supply/value chain optimization [19], and multi-actor distributed processing systems [20].

Other than the profit-allocation concerns mentioned above, it is also of critical importance to examine the viable means for facilitating the desired energy flows among plants in practice. In principle, these flows can be materialized via heat exchange(s) either directly between hot and cold process streams located in different plants or indirectly between the process streams and an intermediate fluid (or the heating and cooling utilities). Although the direct heat exchanges are thermodynamically more efficient than their indirect counterparts, the resulting highly-coupled interplant HEN may pose a control problem in the industrial environment. On the other hand, since the indirect heat integration is facilitated with the auxiliary streams (i.e., steam, cooling water and/or hot oil) that do not take part in any production process, a greater degree of operational flexibility can be achieved [21] and, thus, should be regarded as a more practical alternative.

It should be noted that Cheng et al. [7] considered only the impractical direct heat transfers in their studies and, also, ignored the negotiation powers of the participating plants in their models for allocating the cost savings. To improve the practical feasibility of interplant heat integration projects, the present study aims to modify their sequential optimization approach by replacing the direct interplant heat-transfer options with indirect ones. In addition to the advantage of better operability, the resulting HEN design should also be more acceptable to all players of the game because, on the basis of their respective negotiation powers [11] and the Nash equilibrium constraints [8], the utility cost savings and capital cost increases can both be reasonably distributed among all participating members. Extensive case studies are also presented in this paper to illustrate the proposed procedures and to compare the pros and cons of different indirect heat-exchange alternatives.

Finally, on the basis of the above discussion, the novel contributions of this work can be briefly summarized as follows:

- The profit-allocation concerns in interplant heat integration schemes are addressed systematically with the game theoretic models.
- The more viable means of indirect heat exchanges between the process streams and an intermediate fluid (or the heating and cooling utilities) are considered to facilitate interplant heat flows in practical applications.
- A modified version of the sequential HEN synthesis approach is proposed to incorporate the negotiation powers of the participating plants for allocating their cost savings and, also, to reduce the computation effort to a reasonable level.

2. Sequential optimization procedure

For the sake of illustration clarity, let us briefly review the sequential optimization procedure suggested by Cheng et al. [7]:

i. On the basis of given process data, the minimum acceptable total utility cost of the entire industrial park is determined with a linear program (LP).
ii. By incorporating the constraints of minimum acceptable overall utility cost obtained in step i and also the Nash equilibrium in a nonlinear program (NLP), the heat flows between every pair of plants on site and also their fair trade prices can be calculated accordingly.
iii. By fixing the interplant heat-flow patterns determined in step ii, the minimum total number of both inner- and interplant matches and the corresponding heat duties can be determined with a mixed-integer linear programming (MILP) model.
iv. After constructing a superstructure to facilitate the matches identified in step iii, a nonlinear programming (NLP) model can be formulated to generate the HEN configuration that optimally distributes the total annual cost (TAC) savings among all plants.

This study basically follows the same procedure, while each step is modified for synthesizing the indirect interplant heat integration schemes.

3. Indirect integration via heating and cooling utilities (procedure I)

3.1. Step I_i: determine the minimum total utility cost

If the interplant heat flows can be facilitated only with the heating and cooling utilities, it is necessary to modify the conventional transshipment model [2] for calculating the minimum total utility cost. To facilitate model construction with the standard approach, the entire temperature range should be first partitioned into K intervals according to the initial and target temperatures of all hot and cold streams. Starting from the high temperature end, these intervals are labelled sequentially as $k = 1, 2, \cdots, K$.

Let us assume that a total of P plants take part in the interplant heat integration project and they are labelled as $p = 1, 2, \cdots, P$. Fig. 1 shows the interior and exterior heat-flow patterns of interval k in plant p ($p \neq q$ and $p \neq q'$), and the modified transshipment model can be formulated accordingly as follows:

$$\min \sum_{p=1}^{P} Z_p' \tag{1}$$

s.t.

$$R_{i_p,k} - R_{i_p,k-1} + \sum_{j_p \in C_k^p} Q_{i_p,j_p,k} + \sum_{n_p \in W_k^p} Q_{i_p,n_p,k} + \sum_{\substack{q'=1 \\ q' \neq p}}^{P} \sum_{n_{q'} \in W_k^{q'}} Q_{i_p,n_{q'},k} = Q_{i_p,k}^H, \ i_p \in \tilde{H}_k^p; \tag{2}$$

$$R_{m_p,k} - R_{m_p,k-1} + \sum_{j_p \in C_k^p} Q_{m_p,j_p,k} + \sum_{\substack{q'=1 \\ q' \neq p}}^{P} \sum_{j_{q'} \in C_k^{q'}} Q_{m_p,j_{q'},k} = QS_{m_p,k}, \ m_p \in \tilde{S}_k^p \subset S^p; \tag{3}$$

$$\sum_{i_p \in \tilde{H}_k^p} Q_{i_p,j_p,k} + \sum_{m_p \in \tilde{S}_k^p} Q_{m_p,j_p,k} + \sum_{\substack{q=1 \\ q \neq p}}^{P} \sum_{m_q \in \tilde{S}_k^q} Q_{m_q,j_p,k} = Q_{j_p,k}^C, \ j_p \in C_k^p; \tag{4}$$

$$\sum_{i_p \in \tilde{H}_k^p} Q_{i_p,n_p,k} + \sum_{\substack{q=1 \\ q \neq p}}^{P} \sum_{i_q \in \tilde{H}_k^q} Q_{i_q,n_p,k} = QW_{n_p,k}, \ n_p \in W_k^p \subset W^p; \tag{5}$$

$$QW_{n_p} = \sum_{k=1}^{K} QW_{n_p,k}, \ n_p \in W^p; \tag{6}$$

$$QS_{m_p} = \sum_{k=1}^{K} QS_{m_p,k}, \ m_p \in S^p; \tag{7}$$

$$Z_p' = \sum_{m_p \in S^p} c_{m_p} QS_{m_p} + \sum_{n_p \in W^p} c_{n_p} QW_{n_p}. \tag{8}$$

$$R_{i_p,0} = R_{m_p,0} = R_{i_p,K} = R_{m_p,K} = 0 \tag{9}$$

where, $Q_{i_p,k}^H$ is a model parameter which is used to represent the heat released by hot stream i_p in temperature interval k, $Q_{j_p,k}^C$ is another model parameter which is used to represent the heat absorbed by cold stream j_p in temperature interval k, c_{m_p} and c_{n_p} are model parameters used to denote the unit costs of hot utility m_p and cold utility n_p, respectively, in plant p. Notice also that all other symbols in Equations (1)–(9) are nonnegative variables and they are defined in the Nomenclature section for the sake of brevity.

3.2. Step I_{ii}: set the optimal trade prices

3.2.1. Payoff matrices and strategy vectors

The payoff matrices (denoted as $\mathbf{A}_p$ and $p = 1, 2, \cdots, P$) in the present applications are essentially the same as those used in Cheng et al. [7], while the strategy vectors are more constrained. For illustration clarity, the general structure of the payoff matrices is given below:

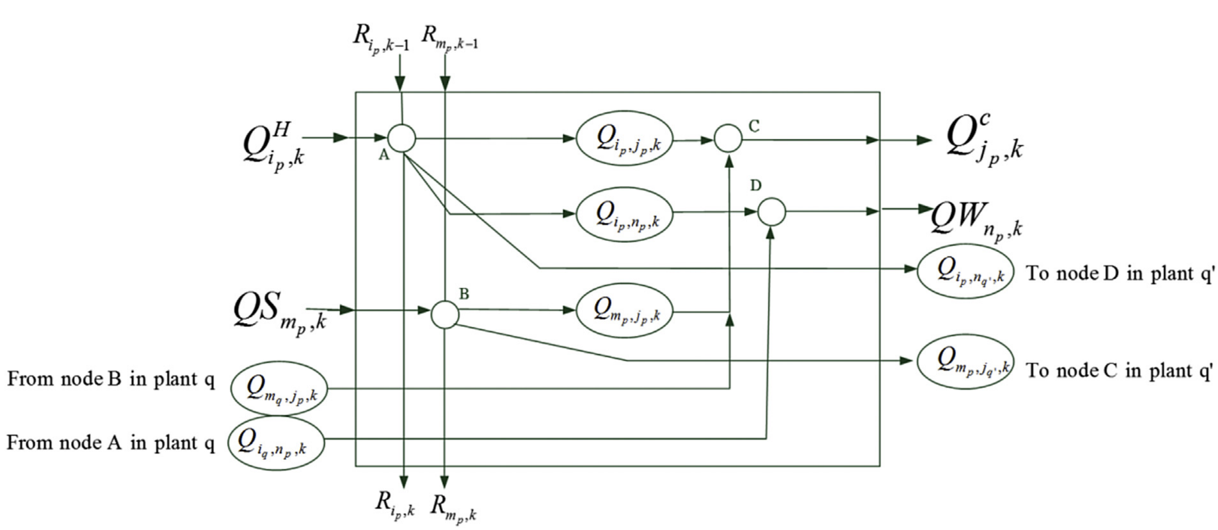

Fig. 1. Interior and exterior heat-flow patterns of interval k in plant$p$ with utility-facilitated interplant heat exchanges.

$$\mathbf{A}_p = \left[\mathbf{A}_{p,q_1}\middle|\mathbf{A}_{p,q_2}\middle|\ldots\middle|\mathbf{A}_{p,q_N}\right] \tag{10}$$

where, $N = P-1$, $q_i \in \{1,2,\cdots,p-1,p+1,\cdots,P\}$ and $i = 1,2,\cdots,N$. Note that each submatrix of $\mathbf{A}_p$ can be explicitly expressed as

$$\mathbf{A}_{p,q_i} = \begin{bmatrix} Payoff^{pUq_iU} & Payoff^{pUq_iL} & NA & NA \\ Payoff^{pLq_iU} & Payoff^{pLq_iL} & NA & NA \\ NA & NA & Payoff^{q_iUpU} & Payoff^{q_iLpU} \\ NA & NA & Payoff^{q_iUpL} & Payoff^{q_iLpL} \end{bmatrix} \tag{11}$$

Note that each element in this submatrix is used to represent the payoff received by plant p per unit heat transferred from plant p to plant q_i or vice versa, while the direction of every feasible heat flow and its source and sink temperatures in relation to their respective pinch points are denoted in the superscript. For example, *payoff* ^{pUq_iL} represents the payoff for transferring a unit of heat from above the pinch in plant p to below the pinch in plant q_i. Note also that the entry *NA* means that the corresponding heat transfer is not allowed since in this case both plants are chosen to be the sources (or sinks). For the other heat flows, the payoffs can be computed according to the following formulas:

$$\begin{aligned} Payoff^{pUq_iU} &= -C_p^{HU} - C_{trd}^{pUq_iU} \\ Payoff^{pUq_iL} &= -C_p^{HU} - C_{trd}^{pUq_iL} \\ Payoff^{pLq_iU} &= +C_p^{CU} - C_{trd}^{pLq_iU} \\ Payoff^{pLq_iL} &= +C_p^{CU} - C_{trd}^{pLq_iL} \end{aligned} \tag{12a}$$

$$\begin{aligned} Payoff^{q_iUpU} &= +C_p^{HU} + C_{trd}^{q_iUpU} \\ Payoff^{q_iLpU} &= +C_p^{HU} + C_{trd}^{q_iLpU} \\ Payoff^{q_iUpL} &= -C_p^{CU} + C_{trd}^{q_iUpL} \\ Payoff^{q_iLpL} &= -C_p^{CU} + C_{trd}^{q_iLpL} \end{aligned} \tag{12b}$$

In the first terms on the right sides of the above equations, C_p^{HU} and C_p^{CU} respectively denote the lowest possible unit costs of heating and cooling utilities in plant p, and both should be positive constants. Note that the sign in front of C_p^{HU} or C_p^{CU} is determined according to the heat flow direction and the pinch location in plant p. Let us consider the first two formulas in Equation (12a) as examples. In particular, *payoff* ^{pUq_iU} and *payoff* ^{pUq_iL} represent the payoffs of transferring a unit of heat from above the pinch in plant p to above and below the pinch in plant q_i respectively. Since the subsystem above the pinch in any process should be considered as a net heat sink, extra heating utility must be consumed by plant p to facilitate either heat flow. Therefore, $-C_p^{HU}$ should be used in these two scenarios to represent the negative contributions to the corresponding payoffs received by plant p. On the other hand, since the 3rd and 4th scenarios in Equation (12a) are concerned with heat flows from below the pinch in plant p to above and below the pinch in plant q_i respectively, both imply that less cooling utility is needed by the subsystem below the pinch (which is a net heat source) in plant p. Consequently, $+C_p^{CU}$ should be used in these two cases to represent the positive contributions to the payoffs of plant p.

The second term of every formula in Equations (12a) And (12b) is the unit trade price of the corresponding heat flow and it is a real-valued variable in the present step. For the sake of formulation consistency, the direction of cash flow is always assigned to be that of the corresponding heat flow. As a result of this model convention, a minus sign is placed in the second term on the right side of each of the first four equations since in these cases plant p pays a fee for delivering heat to plant q_i. Note that the trade prices are treated as variables in this work. A positive price reveals that cash and heat moving toward the same direction, while a negative one denotes otherwise. Since any interplant heat flow simultaneously alters the utility consumptions rates at the source and sink ends, the trade price of this exchange must be bounded by the corresponding utility costs as follows:

$$\begin{aligned} &-\max\left(C_p^{HU}, C_{q_i}^{HU}\right) \le C_{trd}^{pUq_iU} \le -\min\left(C_p^{HU}, C_{q_i}^{HU}\right) \\ &-C_p^{HU} \le C_{trd}^{pUq_iL} \le +C_{q_i}^{CU} \\ &-C_{q_i}^{HU} \le C_{trd}^{pLq_iU} \le +C_p^{CU} \\ &+\min\left(C_p^{CU}, C_{q_i}^{CU}\right) \le C_{trd}^{pLq_iL} \le +\max\left(C_p^{CU}, C_{q_i}^{CU}\right) \end{aligned} \tag{13a}$$

$$\begin{aligned} &-\max\left(C_{q_i}^{HU}, C_p^{HU}\right) \le C_{trd}^{q_iUpU} \le -\min\left(C_{q_i}^{HU}, C_p^{HU}\right) \\ &-C_{q_i}^{HU} \le C_{trd}^{q_iUpL} \le +C_p^{CU} \\ &-C_p^{HU} \le C_{trd}^{q_iLpU} \le +C_{q_i}^{CU} \\ &+\min\left(C_{q_i}^{CU}, C_p^{CU}\right) \le C_{trd}^{q_iLpL} \le +\max\left(C_{q_i}^{CU}, C_p^{CU}\right) \end{aligned} \tag{13b}$$

The more detailed analysis of these inequality constraints can be found in Cheng et al. [6]. On the other hand, the constrained strategy vectors should be determined according to the following interplant heat-exchange rates:

$$QHU_k^{q,p} = \sum_{m_q \in \tilde{S}_k^q} \sum_{j_p \in C_k^p} Q_{m_q,j_p,k} \tag{14}$$

$$QCU_k^{q,p} = \sum_{i_q \in \tilde{H}_k^q} \sum_{n_p \in W_k^p} Q_{i_q,n_p,k} \tag{15}$$

$$QHU_k^{p,q} = \sum_{j_q \in C_k^q} \sum_{m_p \in \tilde{S}_k^p} Q_{m_p,j_q,k} \tag{16}$$

$$QCU_k^{p,q} = \sum_{n_q \in W_k^q} \sum_{i_p \in \tilde{H}_k^p} Q_{i_p,n_q,k} \tag{17}$$

where, $QHU_k^{q,p}$ denotes the total heat-exchange rate between all heating utilities in plant q and all cold streams in interval k of plant p, $QCU_k^{q,p}$ denotes the total heat-exchange rate between all hot streams in interval k of plant q and all cold utilities in plant p, $QHU_k^{p,q}$ denotes the total heat-exchange rate between all heating utilities in plant p and all cold streams in interval k of plant q, $QCU_k^{p,q}$ denotes the total heat-exchange rate between all hot streams in interval k of plant p and all cold utilities in plant q. In order to keep the total utility cost at the minimum level determined in step $\mathrm{I_i}$, these utility-facilitated interplant heat flows should remain unchanged in the present step. As a result, it is only necessary to determine their trade prices.

As mentioned before, four different types of heat exchanges may be selected by plant p and each can be uniquely characterized on the basis of pinch location and the corresponding interplant heat-flow direction. A game strategy in this work can be taken as the ratio between the total amount of a particular type of heat exchanges and that of all possible heat flows in and out of plant p. Specifically,

$$
\begin{aligned}
PR_p^{UO} &= \frac{1}{Q_p^E}\sum_{k\in E_p^U}\sum_{q=1,q\neq p}^{P}\left(QCU_k^{p,q}+QHU_k^{p,q}\right)\\
PR_p^{LO} &= \frac{1}{Q_p^E}\sum_{k\in E_p^L}\sum_{q=1,q\neq p}^{P}\left(QCU_k^{p,q}+QHU_k^{p,q}\right)\\
PR_p^{UI} &= \frac{1}{Q_p^E}\sum_{k\in E_p^U}\sum_{q=1,q\neq p}^{P}\left(QCU_k^{q,p}+QHU_k^{q,p}\right)\\
PR_p^{LI} &= \frac{1}{Q_p^E}\sum_{k\in E_p^L}\sum_{q=1,q\neq p}^{P}\left(QCU_k^{q,p}+QHU_k^{q,p}\right)
\end{aligned}
\tag{18}
$$

where, E_p^U and E_p^L denote the sets of temperature intervals above and below the pinch in plant p, respectively, and $E_p^U \cap E_p^L = \varnothing$. The total volume of energy traffic in and out of plant p should be

$$Q_p^E = \sum_{k=1}^{K}\sum_{q=1,q\neq p}^{P}\left(QHU_k^{p,q}+QCU_k^{p,q}+QHU_k^{q,p}+QCU_k^{q,p}\right) \tag{19}$$

Thus, the strategy vector of plant p can be written as

$$\mathbf{x}_p = \left[\,PR_p^{UO}\quad PR_p^{LO}\quad PR_p^{UI}\quad PR_p^{LI}\,\right]^T \tag{20}$$

3.2.2. Trade prices under Nash equilibrium constraints

The multi-player Nash equilibrium constraints were formulated explicitly by Quintas [22] and they are directly adopted in the present application. Specifically,

$$\mathbf{x}_p^T \sum_{q=1,q\neq p}^{P} \mathbf{A}_{pq}\mathbf{x}_q = \alpha_p \tag{21}$$

$$\sum_{q=1,q\neq p}^{P} \mathbf{A}_{pq}\mathbf{x}_q \le \alpha_p \mathbf{J}_p \tag{22}$$

$$\mathbf{x}_p^T \mathbf{J}_p = 1 \tag{23}$$

$$\mathbf{J}_p = [\,1\quad 1\quad 1\quad 1\,]^T \tag{24}$$

$$\mathbf{x}_p \ge 0 \tag{25}$$

where, $p = 1,2,\cdots,P$, $q = 1,2,\cdots,p-1,p+1,\cdots,P$, α_p is the average total payoff received by plant p, $\mathbf{A}_{pq}$ is the payoff submatrix defined in Equation (11); $\mathbf{x}_p$ and $\mathbf{x}_q$ denote the strategy vectors of plant p and plant q respectively, which can be determined according to Equations (18)–(20).

The following objective function is maximized in a NLP model for setting the proper trade prices in Equation (12):

$$\max \prod_{p=1}^{P}\left(S_p^U\right)^{\varpi_p} \tag{26}$$

where, ϖ_p and S_p^U respectively denote the negotiation power and the total saving of utility cost of plant p They can be computed as follows:

$$\varpi_p = \frac{Z_p'}{\overline{Z}_p} \tag{27}$$

$$S_p^U = \overline{Z}_p - Z_p' + pf_p \tag{28}$$

where, $\overline{Z}_p$ is the minimum total utility cost of a standalone HEN in plant p, Z_p' denotes the minimum total utility cost of plant p achieved with interplant heat integration; pf_p is the total revenue received by plant p via energy trades. The minimum total utility cost of plant p in an interplant heat integration scheme (Z_p') can be determined by solving Equations (1)–(9), while in a standalone HEN this cost $(\overline{Z}_p)$ can be calculated with the conventional transshipment model [2]. Note that the negotiation power of plant p is clearly weakened/strengthened by lowering/raising ϖ_p. In other words, if step $\mathrm{I_i}$ results in significant differences in the utility cost savings, they are moderated in the present step by stipulating proper energy trade prices to maximize the objective function in Equation (26). Finally, the total trade revenue of plant p can be computed according to the following formula:

$$
\begin{aligned}
pf_p = &-\sum_{q'=1,q'\neq p}^{P}\Bigg[C_{trd}^{pUq'U}\sum_{k\in E_p^U\cap E_{q'}^U}\left(QCU_k^{p,q'}+QHU_k^{p,q'}\right)+C_{trd}^{pUq'L}\sum_{k\in E_p^U\cap E_{q'}^L}\left(QCU_k^{p,q'}+QHU_k^{p,q'}\right)\\
&+C_{trd}^{pLq'U}\sum_{k\in E_p^L\cap E_{q'}^U}\left(QCU_k^{p,q'}+QHU_k^{p,q'}\right)+C_{trd}^{pLq'L}\sum_{k\in E_p^L\cap E_{q'}^L}\left(QCU_k^{p,q'}+QHU_k^{p,q'}\right)\Bigg]\\
&+\sum_{q=1,q\neq p}^{P}\Bigg[C_{trd}^{qUpU}\sum_{k\in E_p^U\cap E_q^U}\left(QCU_k^{q,p}+QHU_k^{q,p}\right)+C_{trd}^{qUpL}\sum_{k\in E_p^L\cap E_q^U}\left(QCU_k^{q,p}+QHU_k^{q,p}\right)\\
&+C_{trd}^{qLpU}\sum_{k\in E_p^U\cap E_q^L}\left(QCU_k^{q,p}+QHU_k^{q,p}\right)+C_{trd}^{qLpL}\sum_{k\in E_p^L\cap E_q^L}\left(QCU_k^{q,p}+QHU_k^{q,p}\right)\Bigg]
\end{aligned}
\tag{29}
$$

Notice that the highly-nonlinear function in Equation (26) can be rewritten in the following form to relieve the computational burden:

$$\max \sum_{p=1}^{P} \varpi_p \ln S_p^U \tag{30}$$

The equality and inequality constraints of this optimization problem are given above in Equations (2)–(29). Finally, to make sure that the optimum solution is acceptable to all participating members and also energy efficient, it is necessary to impose a lower bound on the utility cost saving of each plant and set the upper limits of the heating and cooling utility consumption rates. Specifically, the following inequalities are adopted in the proposed model:

$$S_p^U \geq 0 \tag{31}$$

$$\overline{QS}_{m_p} \geq QS_{m_p},\ m_p \in S^p \tag{32}$$

$$\overline{QW}_{n_p} \geq QW_{n_p},\ n_p \in W^p \tag{33}$$

where, the upper limits $\overline{QS}_{m_p}$ and $\overline{QW}_{n_p}$ can be obtained in step $\mathrm{I_i}$ according to Equations (6) and (7).

3.3. Step $\mathrm{I_{iii}}$: identify the optimal matches

The optimization results obtained in the previous two steps can be used for building a MILP model to identify the optimal inner- and inter-plant matches and their heat duties. Specifically, the energy balances given in Equations (2)–(5) and (14)–(17) should all be included in this model as equality constraints. Since the consumption rates of heating and cooling utilities on the right sides of Equations (3) and (5) have already been determined in step $\mathrm{I_i}$, $QS_{m_p,k}$ and $QW_{n_p,k}$ are treated as given parameters in the present step. Similarly, since the consumption rates of heating and cooling utilities on the left sides of Equation (14)–(17) have already been determined in the previous steps and these utilities are used only for the interplant heat exchanges, $QHU_k^{pq}, QCU_k^{pq}, QHU_k^{qp}$, and QCU_k^{qp} should also be considered as given parameters in the present model. The objective function should be

$$\begin{aligned}\min Z = &\sum_{p=1}^{P} \sum_{i_p \in H^p} \sum_{j_p \in C^p} z_{i_p j_p} + \sum_{p=1}^{P} \sum_{i_p \in S^p} \sum_{j_p \in C^p} z_{i_p j_p} + \sum_{p=1}^{P} \sum_{i_p \in H^p} \\ &\times \sum_{j_p \in W^p} z_{i_p j_p} + \sum_{p=1}^{P} \sum_{\substack{q=1\\ q\neq p}}^{P} \sum_{i_p \in S^p} \sum_{j_q \in C^q} z_{i_p j_q} + \sum_{p=1}^{P} \sum_{\substack{q=1\\ q\neq p}}^{P} \sum_{i_p \in H^p} \\ &\times \sum_{j_q \in W^q} z_{i_p j_q}\end{aligned} \tag{34}$$

In addition to the equality constraints mentioned above, the following inequality constraints should also be imposed:

$$Q_{i_p j_q} - z_{i_p j_q} Q_{i_p j_q}^U \leq 0 \tag{35}$$

where, $p, q = 1, 2, \cdots, P$, and $z_{i_p j_q}$ is a binary variable defined as follows

$$z_{i_p j_q} = \begin{cases} 1 & \text{if hot stream(or hot utility)}\, i_p \,\text{exchange heat with cold stream(or cold utility)}\, j_q \\ 0 & \text{otherwise} \end{cases} \tag{36}$$

In the above equations, $Q_{i_p j_q} = \sum_{k \in K} Q_{i_p j_q k}$ denotes the heat duty of match (i_p, j_q) and the model parameter $Q_{i_p j_q}^U$ is its upper bound. It should be noted that, although there are actually seven different types of matches in Fig. 1, $Q_{i_p j_q}$ should be viewed as a generalized notation for representation of the total amount of heat exchanged between hot stream (or utility) i_p and cold stream (or utility) j_q.

3.4. Step $\mathrm{I_{iv}}$: generate the optimal interplant HEN structure

Since only the matches and their duties can be fixed in step $\mathrm{I_{iii}}$, it is necessary to further synthesize the network structure and produce the design specifications of each exchanger. A superstructure-based NLP model can be used for this purpose, e.g., see Floudas et al. [3], and essentially an identical approach is adopted here to build the model constraints. On the other hand, a modified objective function is utilized in this NLP model to facilitate reasonable distribution of the TAC savings of all individual plants (S_p^T), i.e.

$$\max \prod_{p=1}^{P} \left(S_p^T\right)^{\omega_p} \tag{37}$$

or

$$\max \sum_{p=1}^{P} \omega_p \ln S_p^T \tag{38}$$

where, the negotiation power of plant p for the present step, i.e., ω_p, can be determined according to all utility cost savings (S_p^U) as follows

$$\omega_p = \frac{1}{S_p^U}\left(\sum_{p=1}^{P} \frac{1}{S_p^U}\right)^{-1} \tag{39}$$

Notice that, with this formulation, the negotiation power of each plant is inversely proportional to its utility cost saving obtained in step $\mathrm{I_{ii}}$. In other words, the plant with a large utility cost saving may be required to shoulder a large portion of the extra capital investment. On the other hand, the TAC saving of plant p can be computed with the following formula

Table 1
Stream data used in the illustrative example.

Plant #	Stream #	$T_{in}(^\circ C)$	$T_{out}(^\circ C)$	$F_{cp}(\mathrm{kw}/^\circ C)$
P1	H1	150	40	7
	C1	60	140	9
	C2	110	190	8
P2	H1	200	70	5.5
	C1	30	110	3.5
	C2	140	190	7.5
P3	H1	370	150	3.0
	H2	200	40	5.5
	C1	110	360	4.5

$$S_p^T = S_p^U + Af\left(CL_p^I - TC_p^I - \sum_{q'=1,q'\neq p}^{P} TRD_{p,q'} - \sum_{q=1,q\neq p}^{P} TRD_{q,p}\right) \tag{40}$$

where, Af is the annualization factor; CL_p^I denotes the minimum capital cost of a standalone HEN design in plant p, which is treated as a given model parameter in this study; TC_p^I denotes the total capital cost of all inner-plant matches of plant p in an interplant heat integration scheme; $TRD_{p,q'}$ represents the capital cost of interplant match $(i_p, j_{q'})$ shared by plant p and this match is either between a hot stream in plant p and a cold utility in plant q' or between a hot utility in plant p and a cold stream in plant q', $TRD_{q,p}$ also represents the capital cost of interplant match (i_q, j_p) shared by plant p and this match is either between a hot stream in plant q and a cold utility in plant p or between a hot utility in plant q and a cold stream in plant p. The minimum capital cost of a standalone HEN design (CL_p^I) should be computed in advance by following the existing methods, while the other three aforementioned capital costs can be expressed mathematically as follows:

$$TC_p^I = \sum_{i_p \in H^p \cup S^p} \times \sum_{j_p \in C^p \cup W^p} z_{i_p j_p} c_{i_p j_p} \left[\frac{Q_{i_p j_p}}{U_{i_p j_p}\left[\theta^1_{i_p j_p}\theta^2_{i_p j_p}\left(\theta^1_{i_p j_p}+\theta^2_{i_p j_p}\right)/2\right]^{\frac{1}{3}}}\right]^{\beta} \tag{41}$$

$$TRD_{q,p} = \sum_{i_q \in H^q} \times \sum_{j_p \in W^p} z_{i_q j_p}\gamma^p_{i_q j_p} c_{i_q j_p}\left[\frac{Q_{i_q j_p}}{U_{i_q j_p}\left[\theta^1_{i_q j_p}\theta^2_{i_q j_p}\left(\theta^1_{i_q j_p}+\theta^2_{i_q j_p}\right)/2\right]^{\frac{1}{3}}}\right]^{\beta} + \sum_{i_q \in S^q} \times \sum_{j_p \in C^p} z_{i_q j_p}\gamma^p_{i_q j_p} c_{i_q j_p}\left[\frac{Q_{i_q j_p}}{U_{i_q j_p}\left[\theta^1_{i_q j_p}\theta^2_{i_q j_p}\left(\theta^1_{i_q j_p}+\theta^2_{i_q j_p}\right)/2\right]^{\frac{1}{3}}}\right]^{\beta} \tag{43}$$

where, $\gamma^p_{i_p j_{q'}}$ and $\gamma^{q'}_{i_p j_{q'}}$ denote the fractions of capital cost of match $(i_p, j_{q'})$ shared by plant p and plant q' respectively; $\gamma^q_{i_q j_p}$ and $\gamma^p_{i_q j_p}$ denote the fractions of capital cost of match (i_q, j_p) shared by plant q and plant p respectively. Thus, the following equality constraints should also be imposed under the conditions that $p \neq q \neq q'$:

$$\gamma^p_{i_p j_{q'}} + \gamma^{q'}_{i_p j_{q'}} = 1 \tag{44}$$

$$\gamma^q_{i_q j_{p'}} + \gamma^p_{i_q j_p} = 1 \tag{45}$$

Finally, by following the conventional approach [3], a superstructure and the corresponding material and energy balances can be constructed on the basis of the optimization results obtained in step I_{iii}. These constraints should also be included in the NLP model used in the present step for maximizing the objective function defined in Equation (37) or (38).

$$TRD_{p,q'} = \sum_{i_p \in S^p}\sum_{j_{q'} \in C^{q'}} z_{i_p j_{q'}}\gamma^p_{i_p j_{q'}} c_{i_p j_{q'}}\left[\frac{Q_{i_p j_{q'}}}{U_{i_p j_{q'}}\left[\theta^1_{i_p j_{q'}}\theta^2_{i_p j_{q'}}\left(\theta^1_{i_p j_{q'}}+\theta^2_{i_p j_{q'}}\right)/2\right]^{\frac{1}{3}}}\right]^{\beta} + \sum_{i_p \in H^p} \times \sum_{j_{q'} \in W^{q'}} z_{i_p j_{q'}}\gamma^p_{i_p j_{q'}} c_{i_p j_{q'}}\left[\frac{Q_{i_p j_{q'}}}{U_{i_p j_{q'}}\left[\theta^1_{i_p j_{q'}}\theta^2_{i_p j_{q'}}\left(\theta^1_{i_p j_{q'}}+\theta^2_{i_p j_{q'}}\right)/2\right]^{\frac{1}{3}}}\right]^{\beta} \tag{42}$$

Table 2
Utility data used in the illustrative example.

Plant #	Utility Type	Temperature (°C)	Unit Cost (USD/kw·yr)	Upper Limit (kw)
P1	Cooling water	25	10	5000
	High p. steam	200	90	5000
	Fuel oil	500	80	5000
P2	Cooling water	25	22.5	5000
	High p. steam	200	30	5000
	Fuel oil	500	120	5000
P3	Cooling water	25	30	5000
	High p. steam	200	60	5000
	Fuel oil	500	40	5000

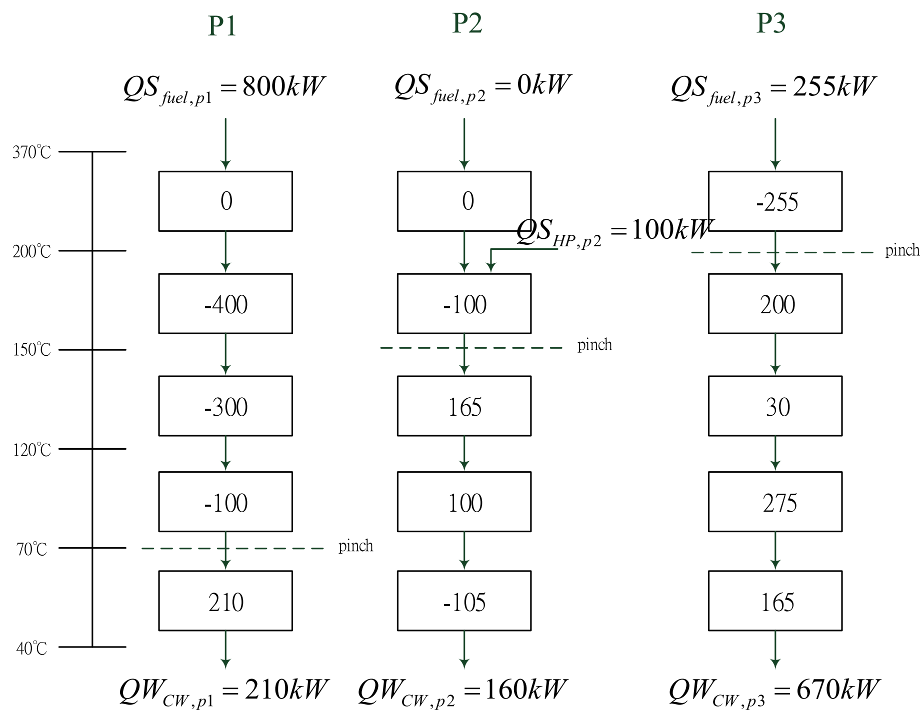

Fig. 2. Standalone heat-flow cascades in the illustrative example.

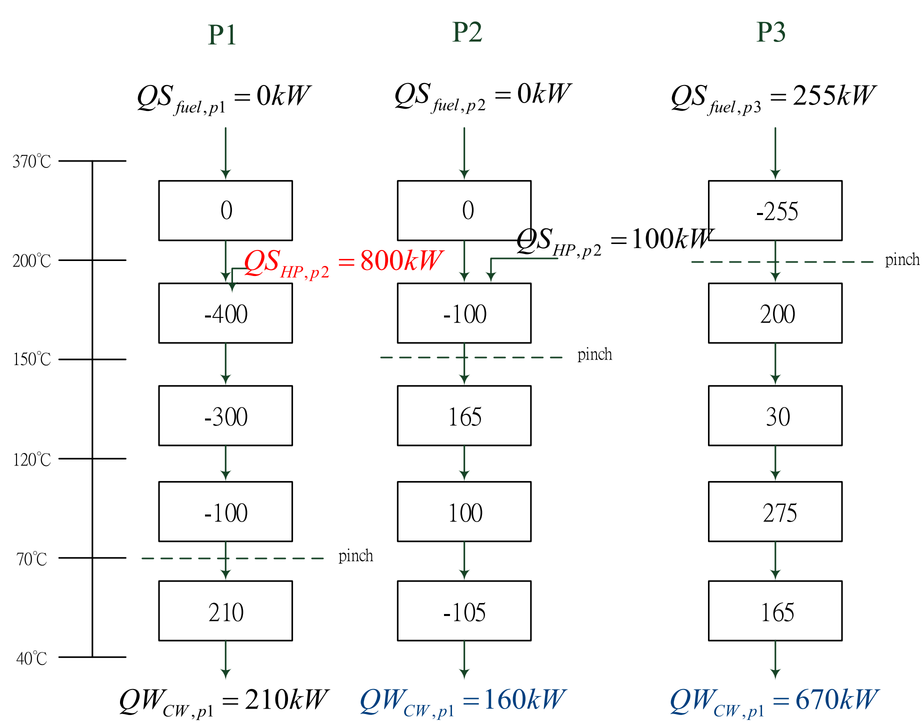

Fig. 3. Integrated heat-flow cascade built at the first step of procedure I in the illustrative example.

3.5. An illustrative example of procedure I

For illustration purpose, the fictitious stream and utility data of three different plants are listed in Tables 1 and 2 respectively [6]. It should be noted from the outset that, since the present study focused primarily upon development of programming models and design procedure, the important issues of data accuracy and the corresponding sensitivity analysis are ignored in this paper for the sake of brevity. On the basis of a minimum temperature approach of 10 K, the three standalone heat-flow cascades can be easily constructed (see Fig. 2) and the corresponding minimum utility costs for P1, P2 and P3 were found to be $66,100$ $(\overline{Z}_{p1})$, $6,600$ $(\overline{Z}_{p2})$ and $30,300$ $(\overline{Z}_{p3})$USD/yr, respectively.

Table 3
Utility cost savings and negotiation powers established at the first step of procedure I in the illustrative example.

Plant #	Utility Type	Utility Costs Without Integration (USD/yr)	Utility Costs after Step I_i(USD/yr)	Total Utility Cost Savings after Step I_i(USD/yr)	Negotiation Powers Gained after Step $I_i(\varpi_p)$
P1	Fuel oil	64,000	0	55,700	0.157
	Cooling water	2100	10,400		
P2	High pres. steam	3000	27,000	−20,400	4.091
	Cooling water	3600	0		
P3	Fuel oil	10,200	10,200	20,100	0.337
	Cooling water	20,100	0		

Table 4
Net utility cost savings and negotiation powers established at the second step of procedure I in the illustrative example.

Plant #	Total Utility Cost Savings after Step I_i(USD/yr)	Total Payoffs from Energy Trades in Step I_{ii}(USD/yr)	Net Utility Cost Savings after Step I_{ii}(USD/yr)	Negotiation Powers Gained after Step $I_{ii}(\omega_p)$
P1	55,700	−43,700	12,000	0.436
P2	−20,400	50,400	30,000	0.174
P3	20,100	−6700	13,400	0.39

Table 5
Optimal matches obtained at the third step of procedure I in the illustrative example.

Hot	Cold					
	P1_C1	P1_C2	P2_C1	P2_C2	P3_C1	P1_CW
P1_H1	1 (560)	0	0	0	0	1 (210)
P2_H1	0	0	1 (280)	1 (275)	0	1 (160)
P3_H1	0	0	0	0	1 (510)	1 (150)
P3_H2	0	0	0	0	1 (360)	1 (520)
P2_HP	1 (160)	1 (640)	0	1 (100)	0	0
P3_Fuel	0	0	0	0	1 (255)	0

3.5.1. *First step of procedure I*

After solving Equations (1)–(9), one can produce the heat-flow cascades in Fig. 3. Note that the lowest-cost heating and cooling utilities are used in all plants to minimize the overall utility cost. The corresponding cost savings and the negotiation powers gained by implementing this step can be summarized in Table 3.

3.5.2. *Second step of procedure I*

This step yields the following fair trade prices of all interplant heat exchanges and the corresponding strategy vectors.

$$C_{trd}^{1U2U}=-30,\ C_{trd}^{1U2L}=0,\ C_{trd}^{1U3U}=-90,\ C_{trd}^{1U3L}=-90,$$
$$C_{trd}^{1L2U}=-30,\ C_{trd}^{1L2L}=+10,\ C_{trd}^{1L3U}=-60,\ C_{trd}^{1L3L}=+30,$$
$$C_{trd}^{2U1U}=-40,\ C_{trd}^{2U1L}=-30,\ C_{trd}^{2U3U}=-30,\ C_{trd}^{2U3L}=-30,$$
$$C_{trd}^{2L1U}=-90,\ C_{trd}^{2L1L}=+10,\ C_{trd}^{2L3U}=-60,\ C_{trd}^{2L3L}=+22.5,$$
$$C_{trd}^{3U1U}=-90,\ C_{trd}^{3U1L}=+10,\ C_{trd}^{3U2U}=-30,\ C_{trd}^{3U2L}=+22.5,$$
$$C_{trd}^{3L1U}=-90,\ C_{trd}^{3L1L}=+10,\ C_{trd}^{3L2U}=0,\ C_{trd}^{3L2L}=+30.$$

$$\begin{bmatrix} PR_1^{UD} \\ PR_1^{LD} \\ PR_1^{UA} \\ PR_1^{LA} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0.491 \\ 0.509 \end{bmatrix},\ \begin{bmatrix} PR_2^{UD} \\ PR_2^{LD} \\ PR_2^{UA} \\ PR_2^{LA} \end{bmatrix}=\begin{bmatrix} 0.833 \\ 0.167 \\ 0 \\ 0 \end{bmatrix},\ \begin{bmatrix} PR_3^{UD} \\ PR_3^{LD} \\ PR_3^{UA} \\ PR_3^{LA} \end{bmatrix}=\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

The resulting utility cost savings and the negotiation powers gained after implementing this step can be summarized in Table 4.

3.5.3. *Third step of procedure I*

The optimal matches obtained in this step are listed in Table 5. In this table, a binary number is given in each cell to denote whether or not a corresponding match should be assigned in the HEN design. A value of 1 indicates that this match is chosen and its heat duty (kW) is specified in a parenthesis in the same cell, while 0 means the corresponding exchanger should be excluded. From Table 5, one can see that there are 7 inner-plant heat exchangers and 5 interplant heat-transfer units.

3.5.4. *Fourth step of procedure I*

On the basis the optimal matches listed in Table 5, one could construct a superstructure for each process stream in the three plants under consideration. The objective function used in the present step, i.e., Equations (37)–(45), can then be formulated according to the negotiation powers given in Table 4, while the other model constraints can be obtained by applying the basic principles of material and energy balances to the superstructures [3]. The annualization factor in Equation (40), i.e., Af, was set at 0.1349. All cost coefficients in Equations (41)–(43) were chosen to be $670\ \mathrm{USD/m^{1.66}}$ and $\beta=0.83$, while all overall heat-transfer coefficients were taken to be $1\ \mathrm{W/m^2\cdot K}$. Finally, the temperature rise of cooling water in every cooler was set to be 5 °C. By solving the corresponding NLP program, one can produce the interplant HEN design presented in Fig. 4 and the final economic assessment in Table 6. This interplant heat integration project should be quite feasible because all TAC savings are positive and reasonably distributed.

4. Indirect integration via intermediate fluid (procedure II)

The mathematical programming models presented in the section can be adopted to identify the proper interplant heat integration schemes by using “hot” oil as the intermediate heat-transfer medium. To simplify model formulations, let us assume that this fluid can only be treated as either a cold or a hot process stream in each plant that joins the integration project and, also, its temperature ranges in all plants must be identical. To facilitate clear explanation, let us introduce two label sets to classify these plants accordingly, i.e., sets PC and PH represent two groups of plants ($PC\cap PH=\varnothing$) and the intermediate fluid is used as a cold stream in the former set and a hot stream in the latter. The optimization steps for the present applications are applied to achieve essentially the same objectives as those given in the previous section, while the model solved in each step should be modified according to the aforementioned assumptions.

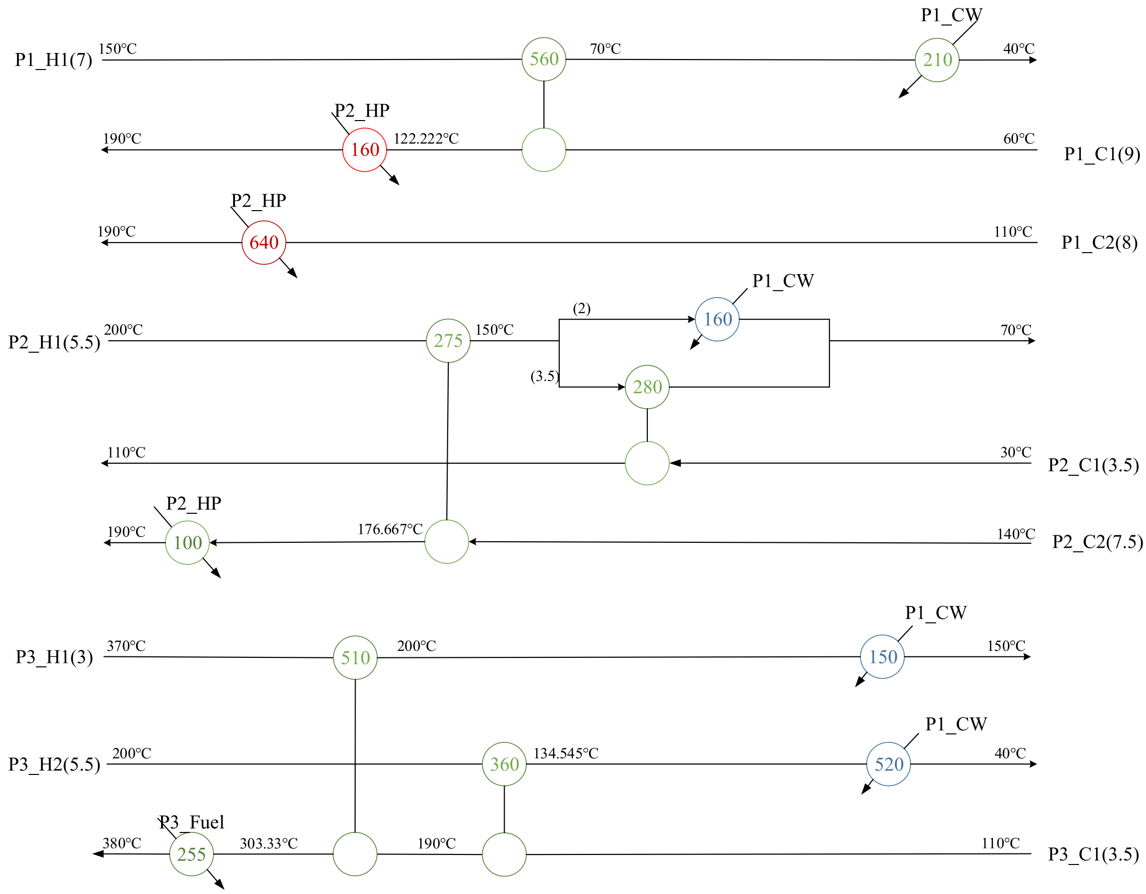

Fig. 4. Interplant HEN design produced at the fourth step of procedure I in the illustrative example.

Table 6
Total annual cost savings achieved with procedure I in the illustrative example.

Plant #	Net Utility Cost Savings after Step I_{ii} (USD/yr)	Total Capital Costs without Integration (USD/yr)	Total Capital Costs after StepI_{iv} (USD/yr)	Total Capital Cost Savings after Step I_{iv} (USD/yr)	TAC Savings after Step I_{iv} (USD/yr)
P1	12,000	5891	7548	−1657	10,343
P2	30,000	5861	7251	−1390	28,610
P3	13,400	7865	7725	140	13,540

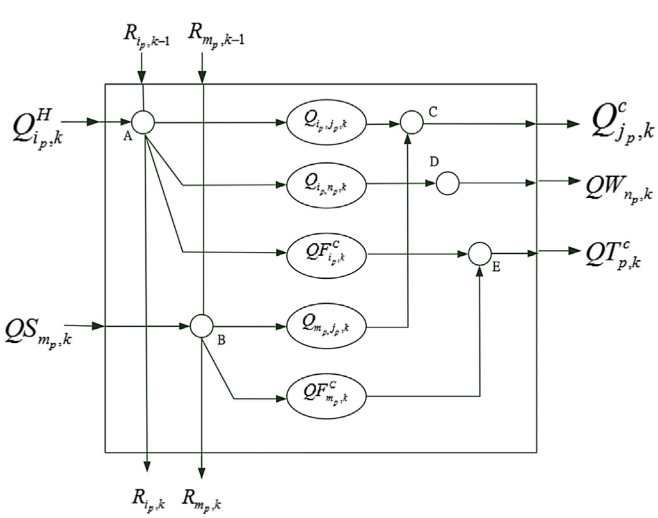

Fig. 5. Interior heat-flow patterns of interval k in plant$p$ with the intermediate fluid acting as a cold stream.

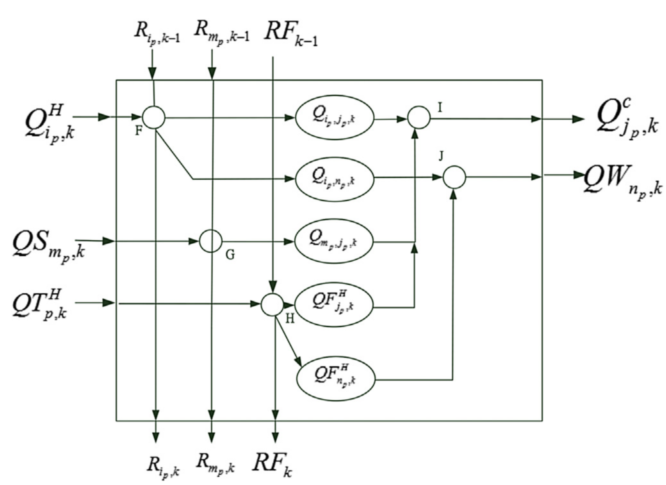

Fig. 6. Interior heat-flow patterns of interval k in plant$p$ with the intermediate fluid acting as a hot stream.

4.1. Step II$_i$: determine the minimum total utility cost

The conventional transshipment model [2] should also be modified for calculating the minimum total utility cost. Fig. 5 shows the interior heat-flow pattern of interval k in plant p if the intermediate fluid is used as a cold stream, while Fig. 6 is its counterpart under the condition that this fluid is a hot stream.

The optimization goal here is the same as that of step I$_i$, i.e., to minimize the total utility cost according to Equation (1), while the equality constraints of the corresponding MINLP model can be obtained primarily on the basis of energy balances. Specifically,

- For $p \in PC$, the energy balances can be established according to Fig. 5 as follows:

A. $$R_{i_p,k} - R_{i_p,k-1} + \sum_{j_p \in C_k^p} Q_{i_p,j_p,k} + \sum_{n_p \in W_k^p} Q_{i_p,n_p,k} + QF^C_{i_p,k} = Q^H_{i_p,k}, \quad i_p \in \tilde{H}_k^p; \tag{46}$$

B. $$R_{m_p,k} - R_{m_p,k-1} + \sum_{j_p \in C_k^p} Q_{m_p,j_p,k} + QF^C_{m_p,k} = QS_{m_p,k}, \quad m_p \in \tilde{S}_k^p; \tag{47}$$

C. $$\sum_{i_p \in \tilde{H}_k^p} Q_{i_p,j_p,k} + \sum_{m_p \in \tilde{S}_k^p} Q_{m_p,j_p,k} = Q^C_{j_p,k}, \quad j_p \in C_k^p; \tag{48}$$

D. $$\sum_{i_p \in \tilde{H}_k^p} Q_{i_p,n_p,k} = QW_{n_p,k}, \quad n_p \in W_k^p; \tag{49}$$

E. $$\sum_{i_p \in \tilde{H}_k^p} QF^C_{i_p,k} + \sum_{m_p \in \tilde{S}_k^p} QF^C_{m_p,k} = FCP_p(T_k - T_{k+1})w_k^{(1)} = QT^C_{p,k}; \tag{50}$$

- For $p \in PH$, the energy balances can be established according to Fig. 6 as follows:

F. $$R_{i_p,k} - R_{i_p,k-1} + \sum_{j_p \in C_k^p} Q_{i_p,j_p,k} + \sum_{n_p \in W_k^p} Q_{i_p,n_p,k} = Q^H_{i_p,k}, \quad i_p \in \tilde{H}_k^p; \tag{51}$$

G. $$R_{m_p,k} - R_{m_p,k-1} + \sum_{j_p \in C_k^p} Q_{m_p,j_p,k} = QS_{m_p,k}, \quad m_p \in \tilde{S}_k^p; \tag{52}$$

H. $$RF_k - RF_{k-1} + \sum_{j_p \in C_k^p} QF^H_{j_p,k} + \sum_{n_p \in W_k^p} QF^H_{n_p,k} = FCP_p(T_k - T_{k+1})w_k^{(1)} = QT^H_{p,k}; \tag{53}$$

I. $$\sum_{i_p \in \tilde{H}_k^p} Q_{i_p,j_p,k} + \sum_{m_p \in \tilde{S}_k^p} Q_{m_p,j_p,k} + QF^H_{j_p,k} = Q^C_{j_p,k}, \quad j_p \in C_k^p; \tag{54}$$

J. $$\sum_{i_p \in \tilde{H}_k^p} Q_{i_p,n_p,k} + QF^H_{n_p,k} = QW_{n_p,k}, \quad n_p \in W_k^p; \tag{55}$$

The following equality constraint must also be imposed to maintain overall material balance on the intermediate fluid:

$$\sum_{p \in PH} FCP_p = \sum_{p \in PC} FCP_p \tag{56}$$

Since the actual temperature range of hot oil in each plant is not a priori given, a binary variable $w_k^{(1)} \in \{0,1\}$ is used in Equations (50) and (53) to signify whether or not interval k exists. Furthermore, to allow this range start and end at any two boundary temperatures of the partitioned intervals, let us introduce two additional binary variables, i.e., $w_k^{(2)}$ and $w_k^{(3)}$, for use in the following constraint:

$$w_k^{(1)} = w_k^{(2)} - w_k^{(3)} \tag{57}$$

where, $k = 1, 2, \cdots, K$. In this equation, $w_k^{(2)} = 1$ means that interval k is higher than the low end of the temperature range, and $w_k^{(3)} = 1$ means that interval k is higher than the corresponding high end. To ensure convexity of the temperature range, two extra logic constraints should be imposed:

$$1 - w_k^{(2)} + w_{k+1}^{(2)} \le 1 \tag{58}$$

$$1 - w_k^{(3)} + w_{k+1}^{(3)} \le 1 \tag{59}$$

where, $k = 1, 2, \cdots, K-1$ and $w_1^{(2)} = 1$. Equations (6) and (7) used in step I$_i$ should also be included in the present model to compute the individual utility consumption rates, but the formulas for calculating the total utility costs should be slightly modified as follows:

$$Z'_p = \sum_{m_p \in S^p} c_{m_p} QS_{m_p} + \sum_{n \in W_p^p} c_{n_p} QW_{n_p} + c_M FCP_p \Delta T_{\min}, \quad p \in PC, \quad M \in S^p; \tag{60}$$

$$Z'_p = \sum_{m_p \in S^p} c_{m_p} QS_{m_p} + \sum_{n \in W_p^p} c_{n_p} QW_{n_p} + c_N FCP_p \Delta T_{\min}, \quad p \in PH, \quad N \in W^p. \tag{61}$$

If the intermediate fluid is adopted to play the role of a cold stream in plant p $(p \in PC)$, its initial and target temperatures must coincide with the aforementioned range in cold-stream temperature. In order to reuse this spent cold stream of plant p as a fresh hot stream in another plant, it is necessary to further raise its temperature in plant p from the cold-stream target to a higher level to achieve an increase of ΔT_{min}. It is also assumed that this temperature change is accomplished by using utility $M \in S^p$ at the unit cost of c_M. Therefore, the third term on the right side of Equation (60) is introduced to account for the corresponding hot utility cost. Similarly, if the intermediate fluid is used as a hot stream in plant p $(p \in PH)$, it is necessary to further lower its temperature from the hot-stream target to achieve an additional decrease of ΔT_{min}, and the third term on the right side of Equation (61) is adopted to account for the corresponding cold utility cost.

Finally, the MINLP model should include the equality constraints to stipulate zero residual heat flows entering the first and leaving the last temperature intervals, i.e., Equation (9) and

$$RF_0 = RF_K = 0 \tag{62}$$

By solving the proposed model, one can obtain the optimal utility consumption rates of every plant (i.e., QS_{m_p} and QW_{n_p}), the heat-capacity flow rate in each plant (FCP_p) and its temperature range ($w_k^{(1)}$). Since all plants must be classified and grouped into two sets, i.e., PC and PH, in advance, it is necessary to construct and solve the proposed MINLP model repeatedly for $2^P - 2$ times, and the optimal solution corresponding to the smallest total utility cost should be chosen for use in the next step.

4.2. Step II$_{ii}$: set the optimal trade prices

The general structure of the payoff matrix in Equation (10) is still valid here, while the submatrix in Equation (11) should be revised as follows:

If $p \in PC$, then the intermediate fluid is only used to facilitate energy export. Thus, $\mathbf{A}_{p,q_i}$ can be reduced to

$$\mathbf{A}_{p,q_i} = \begin{bmatrix} Payoff^{pUq_iU} & Payoff^{pUq_iL} \\ Payoff^{pLq_iU} & Payoff^{pLq_iL} \end{bmatrix} \tag{63}$$

If $p \in PH$, then the intermediate fluid is only used to facilitate energy import. Thus, $\mathbf{A}_{p,q_i}$ can be reduced to

$$\mathbf{A}_{p,q_i} = \begin{bmatrix} Payoff^{q_iUpU} & Payoff^{q_iLpU} \\ Payoff^{q_iUpL} & Payoff^{q_iLpL} \end{bmatrix} \tag{64}$$

As mentioned before, the ratio between the total amount of one particular type of heat exchanges and that of all possible heat flows in and out of plant p is considered as a game strategy in this work. Therefore, Equations (18)–(20) should be reformulated accordingly for the present case. For $p \in PC$, there are two strategies to export heat only, i.e.

$$\begin{aligned} PR_p^{UO} &= \frac{1}{Q_p^{EC}} \sum_{k \in E_p^U} QT_{p,k}^C \\ PR_p^{LO} &= \frac{1}{Q_p^{EC}} \sum_{k \in E_p^L} QT_{p,k}^C \end{aligned} \tag{65}$$

where, $Q_p^{EC} = \sum_{k=1}^{K} QT_{p,k}^C$ and $PR_p^{UO} + PR_p^{LO} = 1$. For $p \in PH$, there are two strategies to import heat only, i.e.

$$\begin{aligned} PR_p^{UI} &= \frac{1}{Q_p^{EH}} \sum_{k \in E_p^U} QT_{p,k}^H \\ PR_p^{LI} &= \frac{1}{Q_p^{EH}} \sum_{k \in E_p^L} QT_{p,k}^H \end{aligned} \tag{66}$$

where, $Q_p^{EH} = \sum_{k=1}^{K} QT_{p,k}^H$ and $PR_p^{UI} + PR_p^{LI} = 1$.

For the purpose of setting the proper trade prices, the general framework of NLP model used here is essentially the same as that described previously in step I$_{ii}$, i.e., see Equations (10), (12), (13), (21)–(28) and (30)–(33). Other than Equations (63)–(66) used in the present model, the only additional change is in computing the total trade revenue received by plant p, i.e., pf_p. Specifically, Equation (29) should be replaced with the two formulas given below:

$$\begin{aligned} pf_p = -Q_p^{EC} \sum_{\substack{q'=1 \\ q' \neq p}}^{P} \lambda_{p,q'}^{O} \Big[& PR_{q'}^{UI} \left(C_{trd}^{pUq'U} PR_p^{UO} + C_{trd}^{pLq'U} PR_p^{LO} \right) \\ & + PR_{q'}^{LI} \left(C_{trd}^{pUq'L} PR_p^{UO} + C_{trd}^{pLq'L} PR_p^{LO} \right) \Big] \end{aligned} \tag{67}$$

where, $p \in PC$, $\lambda_{p,q'}^{O}$ denotes the fraction of total heat exported by plant p that is received by plant $q\prime$, i.e., $\lambda_{p,q'}^{O} = f_{p,q'}/FCP_p$ and $FCP_p = \sum_{q' \in PH} f_{p,q'}$.

$$\begin{aligned} pf_p = Q_p^{EH} \sum_{\substack{q=1 \\ q \neq p}}^{P} \lambda_{q,p}^{I} \Big[& PR_q^{UO} \left(C_{trd}^{qUpU} PR_p^{UI} + C_{trd}^{qUpL} PR_p^{LI} \right) \\ & + PR_q^{LO} \left(C_{trd}^{qLpU} PR_p^{UI} + C_{trd}^{qLpL} PR_p^{LI} \right) \Big] \end{aligned} \tag{68}$$

where, $p \in PH$, $\lambda_{q,p}^{I}$ denotes the fraction of total heat imported by plant p that is released by plant q, i.e., $\lambda_{q,p}^{I} = f_{q,p}/FCP_p$ and $FCP_p = \sum_{q \in PC} f_{q,p}$.

4.3. Step II$_{iii}$: identify the optimal matches

As mentioned previously, the third step of the sequential interplant heat integration procedure is to minimize the total number of matches. In the present case, the objective function should be expressed as

$$\begin{aligned} \min Z = & \sum_{p=1}^{P} \sum_{i_p \in H^p} \sum_{j_p \in C^p} z_{i_p j_p} + \sum_{p=1}^{P} \sum_{i_p \in S^p} \sum_{j_p \in C^p} z_{i_p j_p} + \sum_{p=1}^{P} \sum_{i_p \in H^p} \\ & \times \sum_{j_p \in W^p} z_{i_p j_p} + \sum_{p \in PC} \sum_{i_p \in H^p \cup S^p} zF_{i_p}^C + \sum_{p \in PH} \sum_{j_p \in C^p \cup W^p} zF_{j_p}^H \end{aligned} \tag{69}$$

The first three terms on the right side of this equation are the same as those in Equation (34) and they represent all possible inner-plant matches, while the remaining interplant matches are facilitated indirectly with the intermediate fluid and their heat duties are constrained as follows:

$$QF_{i_p}^C - zF_{i_p}^C UF_{i_p} \le 0, \quad \forall p \in PC \tag{70}$$

$$QF_{j_p}^H - zF_{j_p}^H UF_{j_p} \le 0, \quad \forall p \in PH \tag{71}$$

where, $zF_{i_p}^C \in \{0,1\}$, $zF_{j_p}^H \in \{0,1\}$, $QF_{i_p}^C \left(= \sum_{k \in K} QF_{i_p,k}^C \right)$ denotes the heat duty of the match between hot stream i_p and the intermediate fluid acting as a cold stream; $QF_{j_p}^H$ $(= \sum_{k \in K} QF_{j_p,k}^H)$ denotes the heat duty of the match between cold stream j_p and the intermediate fluid acting as a hot stream; UF_{i_p} and UF_{j_p} respectively denote the upper bounds of the corresponding heat duties.

On the other hand, the model constraints can be formulated with an approach which is very similar to that in step I$_{iii}$. The optimization results obtained previously in steps II$_{i}$ and II$_{ii}$ can be used for building a MILP model to identify the optimal inner- and inter-plant matches and their heat duties. Specifically, the energy balances given in Equations (46)–(56) should be included in this model as equality constraints. Notice that the following model parameters should become available after applying the previous two steps:

- the heating utility consumption rates on the right sides of Equations (47) and (52), i.e., $QS_{m_p,k}$;
- the cooling utility consumption rates on the right sides of Equations (49) and (55), i.e., $QW_{n_p,k}$;
- the heat-capacity flow rate of intermediate medium used in each plant, i.e., FCP_p; ;

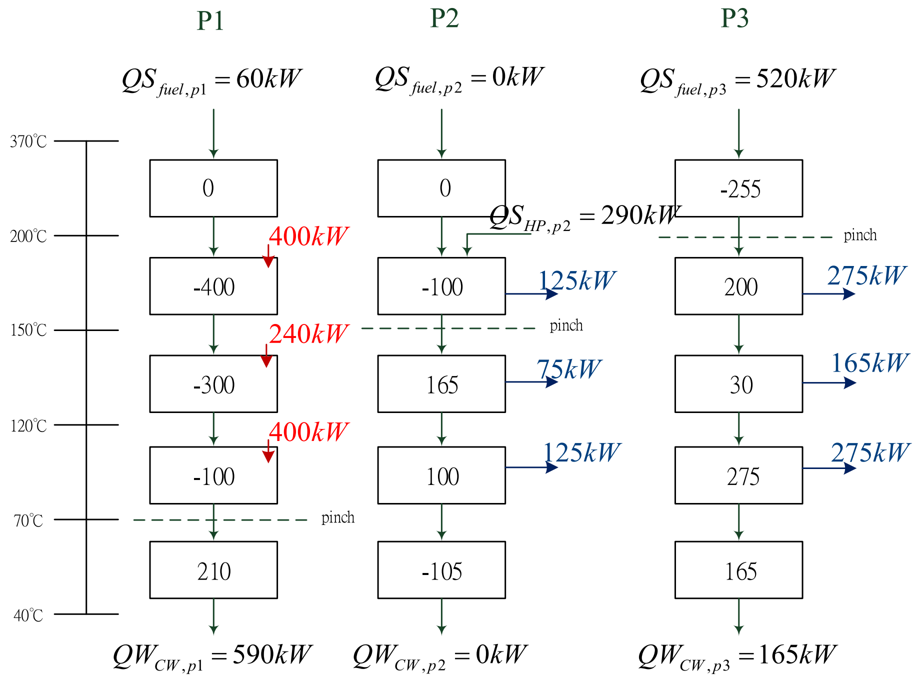

Fig. 7. Integrated heat-flow cascades built at the first step of procedure II in the illustrative example.

Table 7
Utility cost savings and negotiation powers established at the first step of procedure II in the illustrative example.

Plant #	Utility Type	Utility Costs Without Integration $(USD/_{yr})$	Utility Costs after Step $II_i\,(USD/_{yr})$	Total Utility Cost Savings after Step $II_i\,(USD/_{yr})$	Negotiation Powers Gained after Step $II\cdot(\varpi_p)$
P1	Fuel oil	64,000	4800	55,400	0.162
	Cooling water	2100	5900		
P2	High pres. steam	3000	8700	−2100	1.318
	Cooling water	3600	0		
P3	Fuel oil	10,200	20,800	4550	0.850
	Cooling water	20,100	4950		

Table 8
Net utility cost savings and negotiation powers established at the second step of procedure II in the illustrative example.

Plant #	Total Utility Cost Savings after Step $II_i\,(USD/_{yr})$	Total Payoffs from Energy Trades in Step $II_{i1}\,(USD/_{yr})$	Net Utility Cost Savings after Step $II_{i1}\,(USD/_{yr})$	Negotiation Powers Gained after Step $II_{i1}\,(\omega_p)$
P1	55,400	−48,568	6832	0.61
P2	−2100	17,250	15,150	0.27
P3	4550	31,318	35,868	0.12

Table 9
Optimal matches obtained at the third step of procedure II in the illustrative example.

Cold	Hot							
	P1_H1	P1_F	P1_Fuel	P2_H1	P2_HP	P3_H1	P3_H2	P3_Fuel
P1_C1	1 (260)	1 (400)	1 (60)	0	0	0	0	0
P1_C2	0	1 (640)	0	0	0	0	0	0
P1_CW	1 (510)	0	0	0	0	0	0	0
P2_C1	0	0	0	1 (280)	0	0	0	0
P2_C2	0	0	0	1 (110.25)	1 (264.75)	0	0	0
P2_F	0	0	0	1 (325)	0	0	0	0
P3_C1	0	0	0	0	0	1 (660)	0	1 (465)
P3_F	0	0	0	0	0	0	1 (715)	0
P3_CW	0	0	0	0	0	0	1 (165)	0

- the temperature range of intermediate medium used in each plant, i.e., $w_k^{(1)}$.

4.4. Step II$_{iv}$: generate the optimal interplant HEN structure

The objective function of the NLP model used here is the same as that given in Equations (37)–(39), and the corresponding model constraints can be formulated according to the superstructure-based approach suggested by Floudas et al. [3]. As mentioned previously, the above objective function is utilized to facilitate fair distribution of the TAC savings of all plants (S_p^T). If $p \in PC$, the TAC saving can be expressed as

$$S_p^T = S_p^U + Af\left(CL_p^I - TC_p^I - TPC_p^p - \sum_{q \in PH} TPH_q^p\right) \tag{72}$$

Notice that only two variables in this equation have not been defined before, i.e., TPC_p^p and TPH_q^p, and they are the capital costs of interplant heat exchangers (which are facilitated by the intermediate fluid) that must be paid by plant p. The former represents a fraction of the total capital cost of interplant matches in plant p and each can be denoted as (i_p, F), while the latter represents a fraction of the total capital cost of interplant matches in plant q and each is denoted as (F, j_q). On the other hand, the TAC saving of plant $p \in PH$ should be

$$S_p^T = S_p^U + Af\left(CL_p^I - TC_p^I - \sum_{q \in PC} TPC_q^p - TPH_p^p\right) \tag{73}$$

Again in this equation only the definitions of two variables have not been given before. In particular, TPC_q^p and TPH_p^p denote the capital costs of interplant heat exchangers (which are facilitated by the intermediate fluid) that must be paid by plant p. The former is a fraction of the total capital cost of matches in plant q, i.e., (i_q, F), while the latter a fraction of the total capital cost of matches in

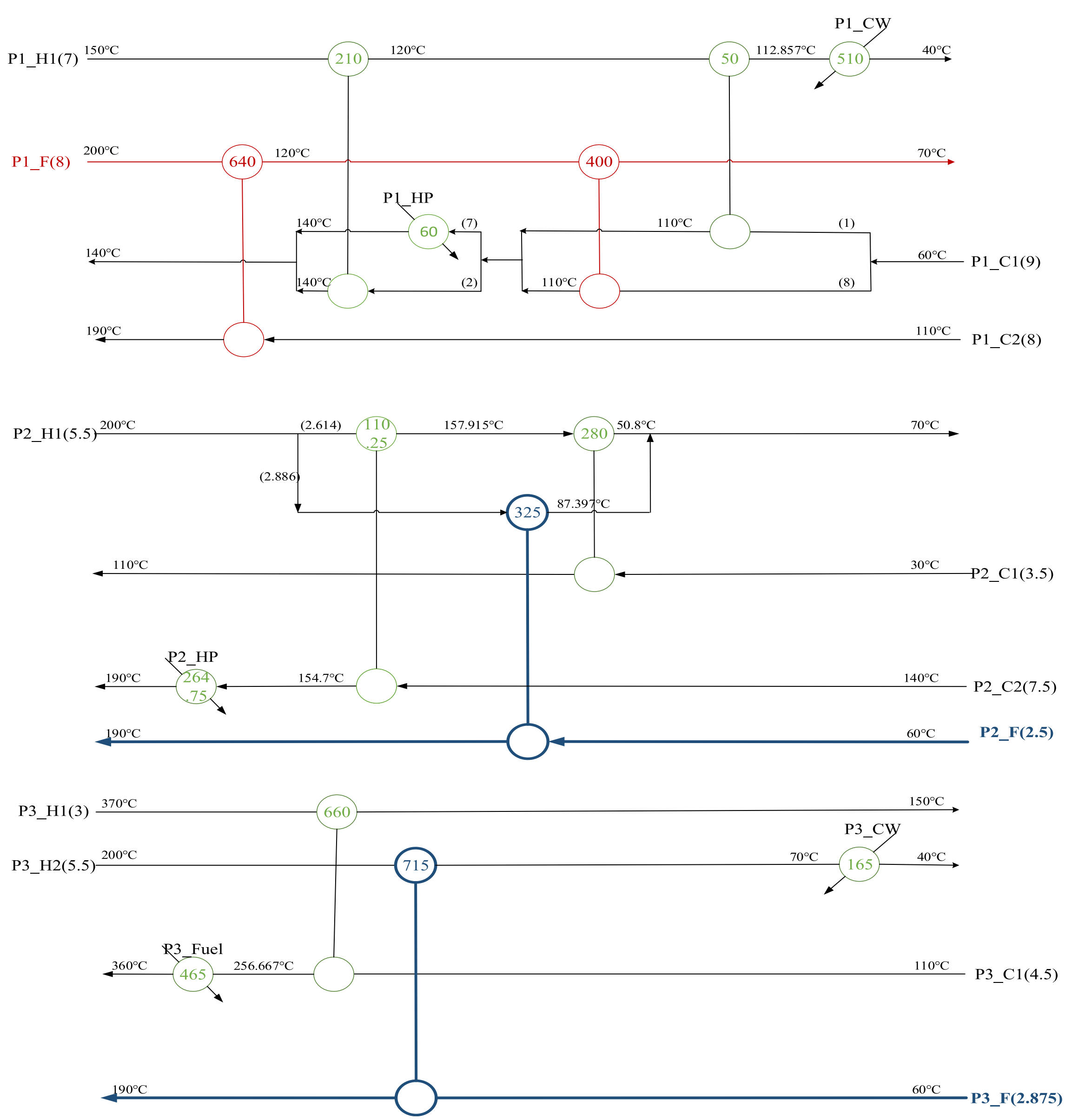

Fig. 8. Interplant HEN design produced at the fourth step of procedure II in the illustrative example.

Table 10
Total annual cost savings achieved with procedure II in the illustrative example.

Plant #	Net Utility Cost Savings after Step II_{i1} (USD/yr)	Total Capital Costs without Integration (USD/yr)	Total Capital Costs after StepII_{iv} (USD/yr)	Total Capital Cost Savings after Step II_{iv} (USD/yr)	TAC Savings after StepII_{iv} (USD/yr)
P1	6832	5891	4273	1618	8450
P2	15,150	5861	12,027	−6166	8984
P3	35,868	7865	8999	−1134	34,734

plant p, i.e., (F, j_p). More specifically, the aforementioned capital costs can be expressed in generalized forms as follows

$$TPH_q^p = \sum_{j_q \in C^q \cup W^q} zF_{j_q}^H \gamma F_{j_q}^p c_{F,j_q} \left[\frac{QF_{j_q}^H}{U_{F,j_q} \left[\theta_{F,j_q}^1 \theta_{F,j_q}^2 \left(\theta_{F,j_q}^1 + \theta_{F,j_q}^2 \right) / 2 \right]^{\frac{1}{3}}} \right]^{\beta} \tag{74}$$

$$TPC_q^p = \sum_{i_q \in H^q \cup S^q} zF_{i_q}^C \gamma F_{i_q}^p c_{i_q,F} \left[\frac{QF_{i_q}^C}{U_{i_q,F} \left[\theta_{i_q,F}^1 \theta_{i_q,F}^2 \left(\theta_{i_q,F}^1 + \theta_{i_q,F}^2 \right) / 2 \right]^{\frac{1}{3}}} \right]^{\beta} \tag{75}$$

where, $p, q \in 1, 2, \cdots, P$; $\gamma F_{j_q}^p$ denotes the fraction of the total capital cost of match (F, j_q) in plant q $(q \in PH)$ that is shared by plant p $(p \in PC)$ and; $\gamma F_{i_q}^p$ denotes the fraction of the total capital cost of match (i_q, F) in plant q $(q \in PC)$ that is shared by plant p $(p \in PH)$. Clearly, additional equality constraints must be imposed upon these cost fractions as follows:

$$\gamma F_{i_p}^p + \sum_{q \in PH} \gamma F_{i_p}^q = 1, \quad \forall p \in PC \tag{76}$$

$$\gamma F_{j_p}^p + \sum_{q \in PC} \gamma F_{j_p}^q = 1, \quad \forall p \in PH \tag{77}$$

where, $\gamma F_{i_p}^p$ in Equation (76) denotes the fraction of the total capital cost of match (i_p, F) in plant p that is shared by plant p itself $(p \in PC)$, while $\gamma F_{i_p}^q$ represents the fraction of the same total capital cost that is shared by plant q, $\gamma F_{j_p}^p$ in Equation (77) denotes the fraction of the total capital cost of match (F, j_p) in plant p that is shared by plant p itself $(p \in PH)$, while $\gamma F_{j_p}^q$ denotes the fraction of the same total capital cost that is shared by plant q. In other words,

- Equation (76) is applicable to match (i_p, F) in plant p when $p \in PC$, and
- Equation (77) is applicable to match (F, j_p) in plant p when $p \in PH$.

Table 11
Stream data used in the additional example.

Plant #	Stream #	$T_{in}(°C)$	$T_{in}(°C)$	F_{cp} (KW/°C)
P1	H1	150	40	7
	C1	60	270	9
	C2	110	190	8
P2	H1	200	70	5.5
	C1	30	110	3.5
	C2	140	190	7.5
P3	H1	380	150	8.5
	H2	200	40	5.5
	C1	110	380	3.5

4.5. Illustrative example of procedure II

Let us again consider the stream and utility data in Tables 1 and 2. Based on a minimum temperature approach of 10 °C, the three individual heat-flow cascades in Fig. 2 and the corresponding minimum utility costs for P1, P2 and P3, i.e., 66,100 ($\overline{Z}_{p1}$), 6,600 ($\overline{Z}_{p2}$) and 30,300 ($\overline{Z}_{p3}$) USD/yr, can still be used for the present application.

4.5.1. First step of procedure II

By repeatedly solving the MINLP model 6 times in step II_i, one can produce the integrated heat-flow cascade in Fig. 7. The temperature range of intermediate fluid is chosen to be [70, 200] °C when it is viewed as a hot stream, while [60, 190] °C if the intermediate fluid is a cold stream. Note that $PC = \{P2, P3\}$ and $PH = \{P1\}$, and it was found that $FCP_{P1} = 8$, $FCP_{P2} = 2.5$ and $FCP_{P3} = 5.5$. Clearly, $FCP_{P1} = FCP_{P2} + FCP_{P3}$. The corresponding cost savings and the negotiation powers gained by implementing this step are summarized in Table 7.

4.5.2. Second step of procedure II

From Fig. 7 and Table 7, the following fair trading prices of interplant heat exchanges can be obtained by solving the proposed NLP model:

$$C_{trd}^{2U1U} = -90.0,\ C_{trd}^{2U1L} = -30.0,$$
$$C_{trd}^{2L1U} = -37.5,\ C_{trd}^{2L1L} = +10.0,$$
$$C_{trd}^{3U1U} = -60.0,\ C_{trd}^{3U1L} = -18.5,$$
$$C_{trd}^{3L1U} = -43.8,\ C_{trd}^{3L1L} = +10.0.$$

The corresponding strategy vectors are:

$$\begin{bmatrix} PR_1^{UI} \\ PR_1^{LI} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} PR_2^{UO} \\ PR_2^{LO} \end{bmatrix} = \begin{bmatrix} 0.385 \\ 0.615 \end{bmatrix}, \begin{bmatrix} PR_2^{UO} \\ PR_2^{LO} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Also, the ratios used in Equations (67) and (68) were found to be: $\lambda_{2,1}^I = 0.3125$, $\lambda_{3,1}^I = 0.6875$ and $\lambda_{3,1}^O = 1.0$. The resulting net utility cost savings and the negotiation powers gained by implementing this step can be summarized in Table 8.

4.5.3. Third step of procedure II

The optimal matches obtained in this step can be found in Table 9. In each cell of this table a binary number is given to denote whether or not a corresponding match is present in HEN design. If a value of 1 is given, then the heat duty (kW) of the corresponding match is also specified in a parenthesis in the same cell. It can be observed from Table 9 that the inner-plant heat exchanges are facilitated with 4 heat exchangers, 2 furnaces, 1 heater (using high pressure steam) and 2 coolers (using cooling water). On the other hand, the interplant heat exchanges are realized with 4 heat-

Table 12
Utility data used in the additional example.

Plant #	Utility Type	Temperature (°C)	Unit Cost (USD/KW·yr)	Upper Limit (KW)
P1	Cooling water	25	10	5000
	High p. steam	400	70	5000
	Medium p. steam	280	50	5000
	Low p. steam	200	40	5000
P2	Cooling water	25	22.5	5000
	High p. steam	400	60	5000
	Medium p. steam	280	40	5000
	Low p. steam	200	25	5000
P3	Cooling water	25	30	5000
	High p. steam	400	80	5000
	Medium p. steam	280	35	5000
	Low p. steam	200	30	5000

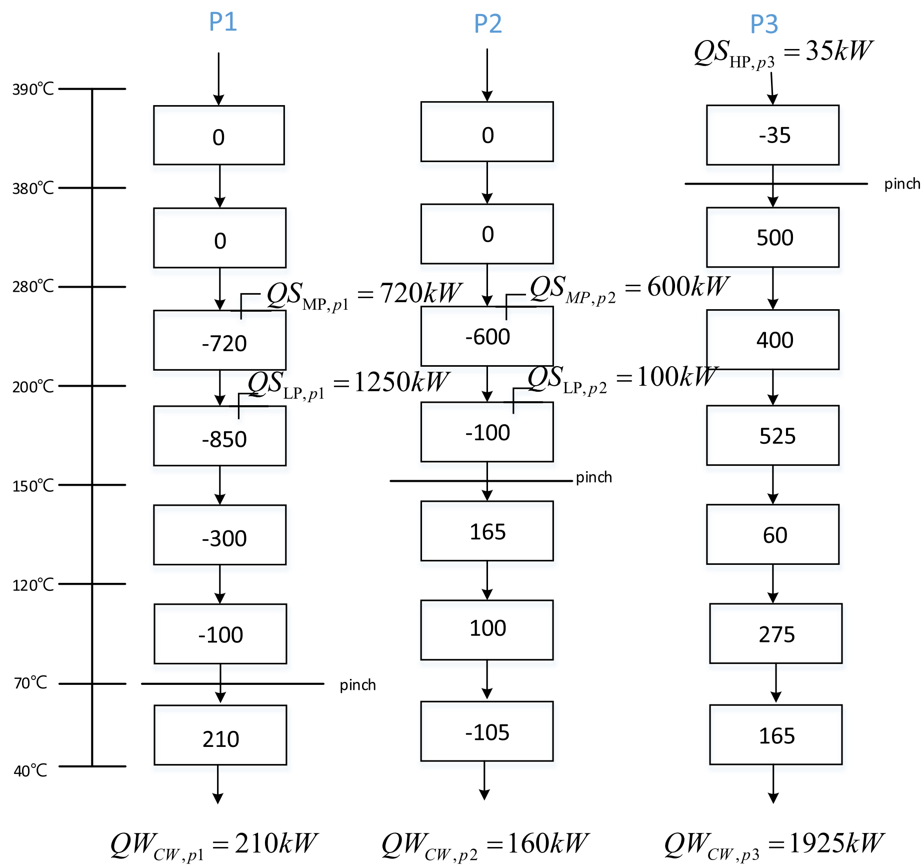

Fig. 9. Standalone heat-flow cascades in the additional example.

transfer units. The intermediate fluid is used as the cold stream in one such unit in plant P2 and also in another in plant P3, while it is the hot stream in two separate units in plant P1.

4.5.4. *Fourth step of procedure II*

On the basis the optimal matches listed in Table 9, a unique superstructure can be built for every process stream in the three plants under consideration. The objective function defined by Equations (37), (39), (41) and (72)–(75) can then be established according to the negotiation powers given in Table 8, while the other model constraints can be obtained by applying the basic principles of material and energy balances to the superstructures [3]. The annualization factor in Equation (73), i.e., Af, was also set at 0.1349. All cost coefficients in Equations (41), (74) and (75) were chosen to be 670 USD/m$^{1.66}$ and $\beta = 0.83$, while all overall heat-transfer coefficients were taken to be 1 W/m^2·K. Finally, the temperature rise of cooling water in every cooler was set to be 5 °C.

By solving the corresponding NLP program, one could generate the interplant HEN design presented in Fig. 8 and the final economic assessment in Table 10. It can be observed that, although the capital investments of plants P2 and P3 must be increased to realize the required inner- and inter-plant heat exchanges, this interplant heat integration project should still be quite feasible since the TAC savings of all plants are positive and reasonably distributed.

5. Case studies

By comparing the net savings in utility costs (see Tables 4 and 8) achieved in the aforementioned examples and those reported previously in Cheng et al. [6], i.e., 74,450 USD/yr, one could

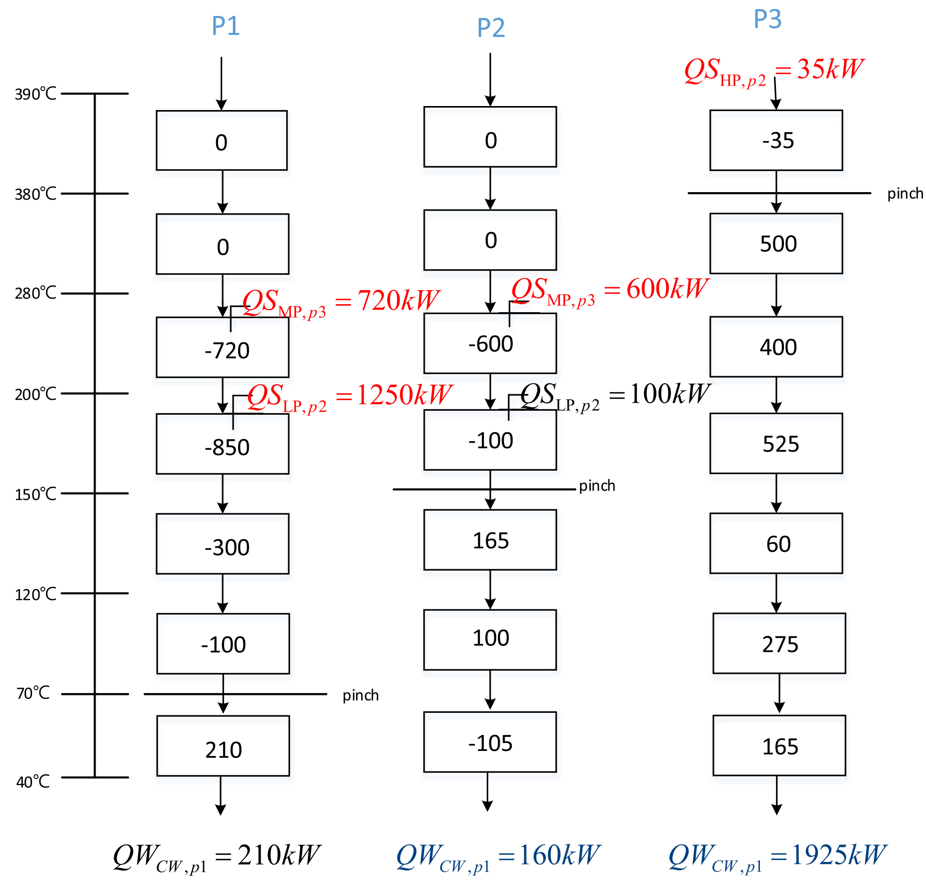

Fig. 10. Integrated heat-flow cascades built at the first step of procedure I in the additional example.

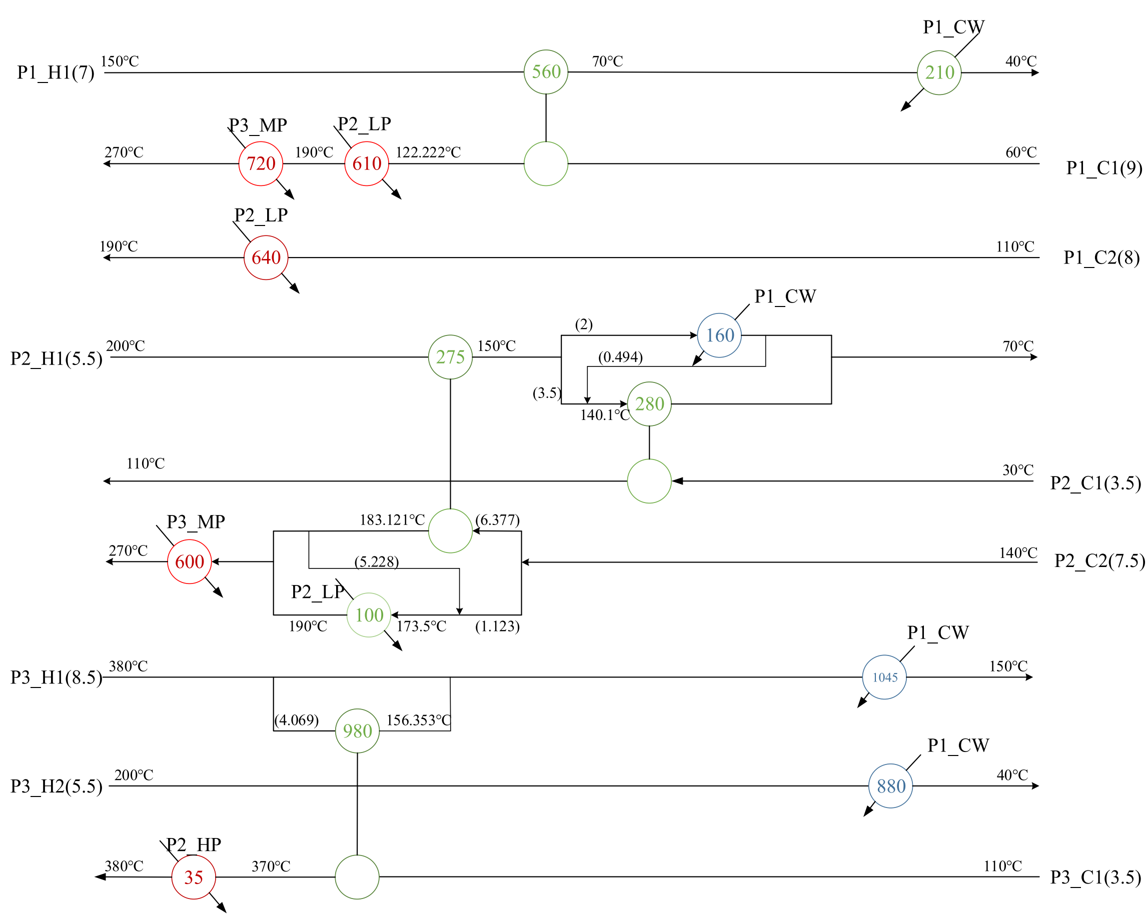

Fig. 11. Interplant HEN design produced with procedure I in the additional example.

Table 13
Total annual cost savings achieved with procedure I in the additional example.

Plant #	Net Utility Cost Savings after Step I_{ii} (USD/yr)	Total Capital Costs without Integration (USD/yr)	Total Capital Costs after Step I_{iv} (USD/yr)	Total Capital Cost Savings after Step I_{iv} (USD/yr)	TAC Savings after Step I_{iv} (USD/yr)
P1	9748	6066	8367	−2500	7248
P2	38,783	6321	8152	−1882	36,901
P3	25,219	4533	6674	−1363	23,856

conclude that in general the overall energy cost of a direct heat integration scheme should be lower than any of its indirect counterpart and, also, the sum of net utility cost savings achieved with an intermediate fluid should be larger than that with the heating and cooling utilities. From Figs. 2 and 3, it can be observed that the minimum consumption rates of steams and cooling water of the standalone plants are basically the same as those of the utility-facilitated indirect heat integration scheme. Since every available hot/cold utility in the latter case is essentially a common heat source/sink shared by all plants, the total utility cost saving reported in Table 3, i.e., $55,400\ (=55,700-20,400+20,100)$ USD/yr, is achieved in step I_i simply by replacing the utilities used in standalone cascades (Fig. 2) with their cheapest alternatives (Fig. 3). One can also see from Figs. 3 and 7 that, instead of sharing utilities across the plant boundaries, the indirect interplant heat integration scheme can be made more efficient by using an intermediate fluid to provide better heat-exchange opportunities. Furthermore, from Figs. 7 and 3 in Cheng et al. [6], one would expect that a direct interplant heat integration scheme can be adopted to reap the largest possible energy saving as long as the required interplant heat exchanges are realizable in practice. Finally, in the illustrative example discussed above, the total capital cost of the HEN synthesized with procedure I is lower than that with procedure II and, also, the former is structurally simpler and should be considered as a more operable and controllable design.

An extra example is presented below to further confirm the above findings:

5.1. Process data

Tables 11 and 12 respectively show the stream and utility data used in an additional example. Based on a minimum temperature approach of 10 K, the corresponding standalone heat-flow cascades can be built and they are given in Fig. 9. The minimum utility costs of P1, P2 and P3 can be determined respectively to be $88,100\ (\overline{Z}_{p1}), 30,100\ (\overline{Z}_{p2}),$ and $60,500\ (\overline{Z}_{p3})$, USD/yr.

5.2. Procedure I

The integrated heat-flow cascade given in Fig. 10 was produced at the first step of procedure I. Although the required utility consumption rates are the same as those in the standalone heat-flow cascades (see Fig. 9), the cheapest utility is adopted at every

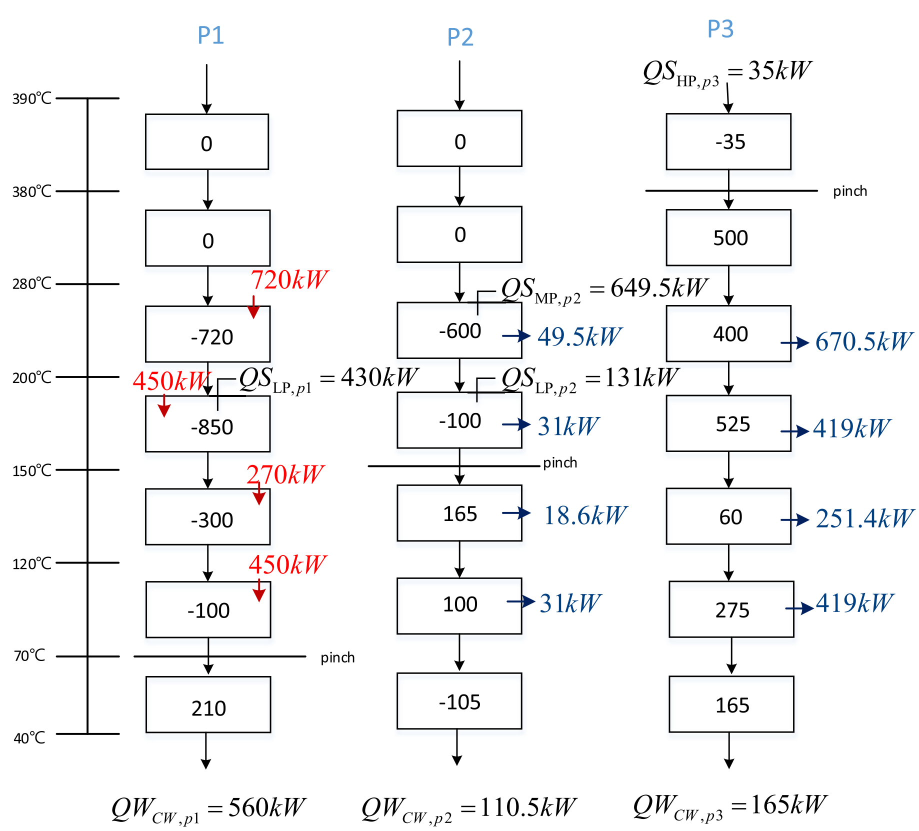

Fig. 12. Integrated heat-flow cascades built at the first step of procedure II in the additional example.

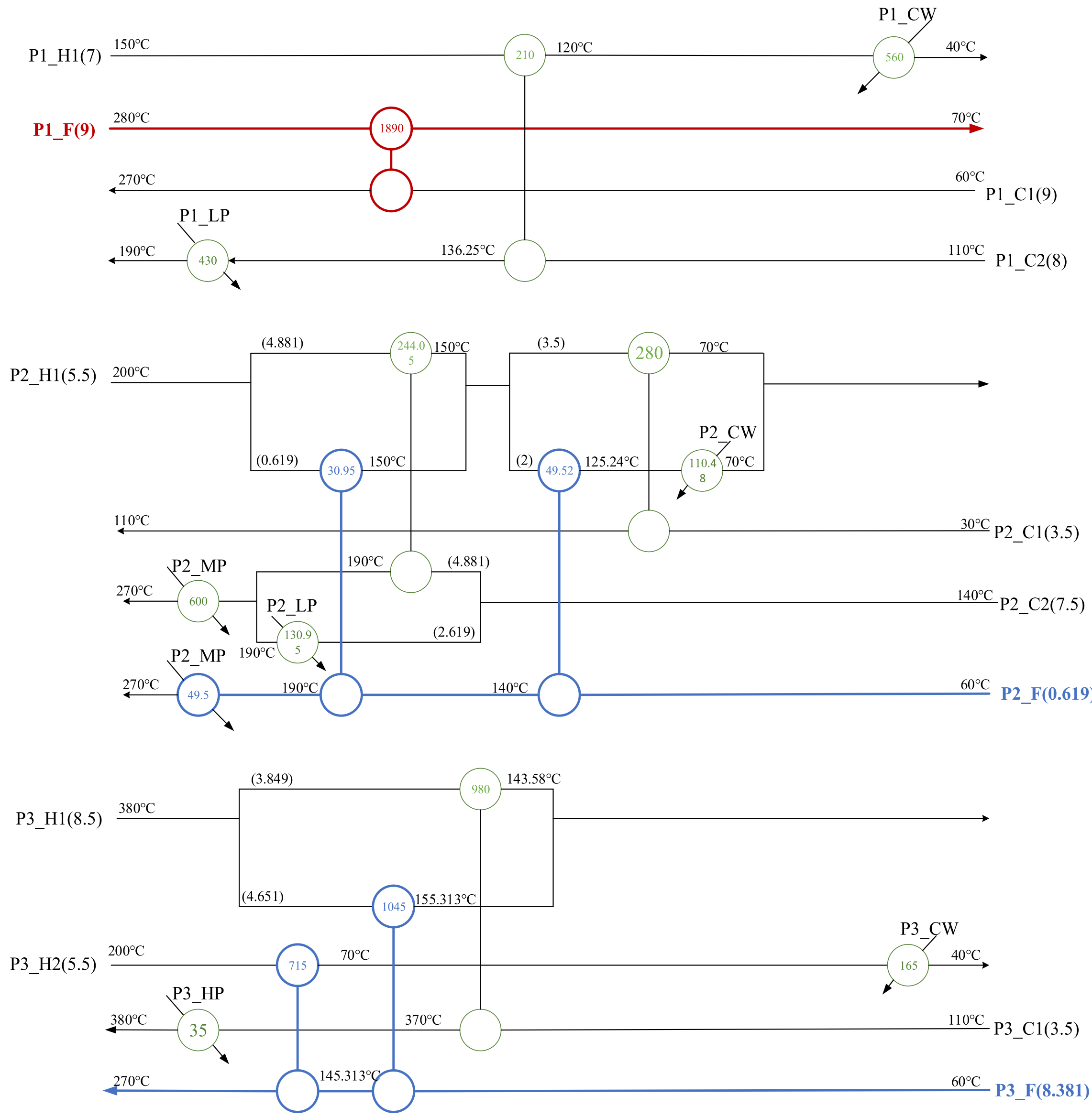

Fig. 13. Interplant HEN design produced with procedure II in the additional example.

occasion in this integrated scheme, i.e., the cooling water is always taken from P1, the high- and low-pressure steams are from P2, and the medium-pressure steam is from P3. The resulting interplant HEN design is presented in Fig. 11, while a complete economic summary is provided in Table 13.

5.3. Procedure II

The integrated heat-flow cascade in Fig. 12 was produced by applying the first step of procedure II. The temperature range of hot oil in this case can be chosen to be $[70,\ 280]\ ^\circ C$ if it is used as a hot stream, while $[60,\ 270]\ ^\circ C$ if otherwise. Note that $PC = \{P2,\ P3\}$ and $PH = \{P1\}$, and it was determined that $FCP_{P1} = 9$, $FCP_{P2} = 0,619$ and $FCP_{P3} = 8.381$. Note that, by using the intermediate fluid, a significant amount of 1459.5 kW can be reduced respectively from the hot and cold utility consumption rates of the integrated scheme obtained with procedure I (see Figs. 10 and 12). By adopting the same parameter values presented previously in subsection 3.5.4 or 4.5.4, one can produce the interplant HEN design in Fig. 13 and the corresponding cost summary can be found in Table 14.

Table 14
Total annual cost savings achieved with procedure II in the additional example.

Plant #	Net Utility Cost Savings after Step II_{i1} ($USD/_{yr}$)	Total Capital Costs without Integration ($USD/_{yr}$)	Total Capital Costs after StepII_{iv} ($USD/_{yr}$)	Total Capital Cost Savings after Step I_{iv} ($USD/_{yr}$)	TAC Savings after Step II_{iv} ($USD/_{yr}$)
P1	73,208	6066	11,841	−5775	67,433
P2	6965	6321	7096	−775	6190
P3	36,237	4533	4691	−158	36,079

5.4. Cost analysis

From Tables 13 and 14, it can be observed that these optimization results clearly reaffirm the conclusions drawn from the previous example (see the beginning of Section 5). Notice that the total utility cost saving achieved with the heating and cooling utilities $(73,750\ \mathrm{USD/yr})$ is much smaller than that with the intermediate fluid $(116,410\ \mathrm{USD/yr})$. This difference is probably due to their roles in facilitating heat flows between plants. Although various hot and cold utilities may be available in different plants, they are all supposed to be treated equally without distinguishing their origins. Thus, if the interplant heat exchanges are facilitated with utilities, then the total hot and cold utility consumption rates in every plant should respectively remain unchanged. A considerable utility cost saving may still be achievable in this scenario due to the use of cheaper alternatives. On the other hand, notice that the intermediate fluid is treated as either an *extra* hot or cold process stream in every individual plant. These additional streams are selected in step $\mathrm{II_i}$ for optimally adjusting the grand composite curve of each plant so as to make the corresponding total utility cost lower than that of the standalone counterpart. As a result, the overall consumption rates of hot and cold utilities of the entire site can both be reduced to lower levels as well.

However, since the intermediate fluid is present in a much wider temperature range than that of any hot or cold utility, the interplant HEN design synthesized with the intermediate fluid requires a slightly higher total capital cost than the utility-facilitated design. This tendency is also consistent with that found in the previous example. Finally, note that the eventual feasibility of each interplant heat integration program can also be ensured by the reasonably distributed TAC savings among all participating members.

6. Conclusions

This study aims to improve the practical applicability of interplant heat integration scheme. An existing game-theory based sequential optimization strategy [6] has been improved in this work on the basis of the individual negotiation power of every participating plant to stipulate the “fair” price for each energy trade, to determine the “reasonable” proportions of capital cost to be shouldered by the involved parties of every interplant heat exchanger, and to produce an acceptable distribution of TAC savings. Also, to address various safety and operational concerns against direct heat transfers across plant boundaries, the interplant heat flows have been facilitated in the proposed integration schemes with either the available utilities or an extra intermediate fluid. Because of these additional practical features in interplant heat integration, realization of the resulting financial and environmental benefits should become more likely. Furthermore, from the optimization results obtained in case studies, one can find that feasible schemes can indeed be synthesized with the above two procedures. The interplant HEN design generated with procedure I should be simpler and more operable, while its counterpart created by procedure II is usually more energy efficient.

Nomenclature

Abbreviations

HEN	heat exchanger network
LP	linear program
MILP	mixed-integer linear program
MINLP	mixed-integer nonlinear programming
NA	the corresponding heat transfer is not allowed
NLP	nonlinear programming
TAC	total annual cost
TSHI	total site heat integration

Sets

C_k^p	the set of cold streams in interval k of plant p
C^p	the set of cold streams in plant p
$\mathrm{E_p^U}$	the set of temperature intervals above the pinch in plant p
$\mathrm{E_p^L}$	the set of temperature intervals below the pinch in plant p
H_k^p	the set of hot process streams in interval k of plant p
H^p	the set of hot process streams in plant p
$\tilde{H}_k^p$	the set of hot process streams in or above interval k of plant p
PC	the set of plants in which the intermediate fluid is treated as a cold stream
PH	the set of plants in which the intermediate fluid is treated as a hot stream
S	the set of all hot utilities
S^p	the set of hot utilities in plant p
S_k^p	the set of hot utilities in interval k of plant p
$\tilde{S}_k^p$	the set of hot utilities in or above interval k of plant p
W	the set of all cold utilities
W^p	the set of cold utilities in plant p
W_k^p	the set of cold utilities in interval k of plant p

Superscripts

H	hot stream
C	cold stream
pUq_iL	from above the pinch in plant p to below the pinch in plant q_i
pUq_iU	from above the pinch in plant p to above the pinch in plant q_i

Indices

k	label of temperature interval k ($k = 1, 2, \cdots, K$)
p	label of plant $p(q^{'} = 1, 2, \cdots, P)$
q	label of plant $q(q^{'} = 1, 2, \cdots, P)$
q'	label of plant $q'(q^{'} = 1, 2, \cdots, P)$
i_p	label of hot process stream i_p in plant p.
j_p	label of cold process streamj_p in plant p.
m_p	label of hot utility m_p in plant p.
n_p	label of cold utility n_p in plant p.

Parameters

Af	the annualization factor
c_{i_p,j_q}	the coefficient in the capital cost model of heat exchanger between hot stream i_p and cold stream j_q $(\mathrm{USD/m^{1.66}})$
$c_{i_p,j_{q'}}$	the coefficient in the capital cost model of heat exchanger between hot stream i_p and cold stream j_q $(\mathrm{USD/m^{1.66}})$
c_{F,j_q}	the coefficient in the capital cost model of heat exchanger between intermediate fluid F and cold stream j_q $(\mathrm{USD/m^{1.66}})$
$c_{i_q,F}$	the coefficient in the capital cost model of heat exchanger between hot stream i_q and intermediate fluid F $(\mathrm{USD/m^{1.66}})$

C_p^{HU} the lowest unit cost of hot utility needed in plant p to facilitate the corresponding interplant heat flow (USD/kW·y)
C_p^{CU} the lowest unit cost of cold utility needed in plant p to facilitate the corresponding interplant heat flow (USD/kW·y)
c_{m_p} the unit cost of hot utility m_p (USD/kW·y)
c_{n_p} the unit cost of cold utility n_p (USD/kW·y)
K the total number of temperature intervals
P the total number of plants taking part in the interplant heat integration project
$Q_{i_p,k}^H$ the heat released by hot stream i_p in temperature interval k (kW)
$Q_{j_p,k}^C$ the heat absorbed by cold stream j_p in temperature interval k (kW)
$\overline{QS}_{m_p}$ the upper bound of the consumption rate of hot utility m_p (kW)
$\overline{QW}_{n_p}$ the upper bound of the consumption rate of cold utility n_p (kW)
U_{i_p,j_q} the overall heat transfer coefficient between hot stream i_p and cold stream j_q (kW/m^2·K)
$U_{i_p,j_{q'}}$ the overall heat transfer coefficient between hot stream i_p and cold stream $j_{q'}$ (kW/m^2·K)
U_{i_q,j_p} the overall heat transfer coefficient between hot stream i_q and cold stream j_p (kW/m^2·K)
U_{F,j_q} the overall heat transfer coefficient between hot oil F and cold stream j_p (kW/m^2·K)
$U_{i_q,F}$ the overall heat transfer coefficient between hot oil F and cold stream j_p (kW/m^2·K)
ΔT_{min} the minimum temperature approach (°C)

Variables

$\mathbf{A}_p$ the payoff matrix of plant p ($p = 1, 2, \cdots, P$)
$\mathbf{A}_{p,q_i}$ a sub-matrix of the payoff values between plant p and plant q_i
$C_{trd}^{pUq_iU}$ the trade price paid by plant p for transferring a unit of heat from a temperature above the pinch point of plant p to a temperature above the pinch point of plant q_i (USD/kW·y)
$C_{trd}^{pUq_iL}$ the trade price paid by plant p for transferring a unit of heat from a temperature above the pinch point of plant p to a temperature below the pinch point of plant q_i (USD/kW·y)
$C_{trd}^{pLq_iU}$ the trade price paid by plant p for transferring a unit of heat from a temperature below the pinch point of plant p to a temperature above the pinch point of plant q_i (USD/kW·y)
$C_{trd}^{pLq_iU}$ the trade price paid by plant p for transferring a unit of heat from a temperature below the pinch point of plant p to a temperature below the pinch point of plant q_i (USD/kW·y)
$C_{trd}^{q_iUpU}$ the trade price paid by plant q_i for transferring a unit of heat from a temperature above the pinch point of plant q_i to a temperature above the pinch point of plant p (USD/kW·y)
$C_{trd}^{q_iLpU}$ the trade price paid by plant q_i for transferring a unit of heat from a temperature below the pinch point of plant q_i to a temperature above the pinch point of plant p (USD/kW·y)
$C_{trd}^{q_iUpL}$ the trade price paid by plant q_i for transferring a unit of heat from a temperature above the pinch point of plant q_i to a temperature below the pinch point of plant p (USD/kW·y)
$C_{trd}^{q_iLpL}$ the trade price paid by plant q_i for transferring a unit of heat from a temperature below the pinch point of plant q_i to a temperature below the pinch point of plant p (USD/kW·y)
CL_p^I the minimum total capital cost of a standalone HEN design in plant p (USD/y)
$Payoff^{pLq_iL}$ the unit payoff value of exporting heat at a temperature below the pinch in plant p to a temperature below the pinch in plant q_i (USD/kW·y)
$Payoff^{q_iUpU}$ the unit payoff value of exporting heat at a temperature above the pinch in plant q_i to a temperature above the pinch in plant p (USD/kW·y)
$Payoff^{q_iLpU}$ the unit payoff value of exporting heat at a temperature below the pinch in plant q_i to a temperature above the pinch in plant p (USD/kW·y)
$Payoff^{q_iUpL}$ the unit payoff value of exporting heat at a temperature above the pinch in plant q_i to a temperature below the pinch in plant p (USD/kW·y)
$Payoff^{q_iLpL}$ the unit payoff value of exporting heat at a temperature below the pinch in plant q_i to a temperature below the pinch in plant p (USD/kW·y)
pf_p the total revenue received by plant p via energy trades (USD/y)
$Q_{i_p,j_p,k}$ the amount of heat exchanged between hot stream i_p and cold stream j_p in interval k of plant p (kW)
$Q_{i_p,n_p,k}$ the amount of heat exchanged between hot stream i_p and cold utility n_p in interval k of plant p (kW)
$Q_{i_p,n_{q'},k}$ the amount of heat exchanged between hot stream i_p in interval k and cold utility $n_{q'}$ of plant q' (kW)
$Q_{m_p,j_p,k}$ the amount of heat exchanged between hot utility m_p and cold stream j_p in interval k of plant p (kW)
$Q_{m_p,j_{q'},k}$ the amount of heat exchanged between hot utility m_p in interval k of plant p and cold stream $j_{q'}$ of plant q' (kW)
$Q_{m_q,j_p,k}$ the amount of heat exchanged between hot utility m_q in plant q and cold stream j_p in interval k of plant p (kW)
Q_p^E the total volume of energy traffic in and out of plant p (kW)
$QCU_k^{q,p}$ the total heat-exchange rate between all hot streams in interval k of plant q and all cold utilities in plant p (kW)
$QCU_k^{p,q}$ the total heat-exchange rate between all hot streams in interval k of plant p and all cold utilities in plant q (kW)
$QF_{i_p,k}^C$ the amount of heat exchanged between hot stream i_p and hot oil in interval k of plant p (kW)
$QF_{m_p,k}^C$ the amount of heat exchanged between hot utility m_p and hot oil in interval k of plant p (kW)
$QF_{j_p,k}^H$ the amount of heat exchanged between hot oil and cold stream j_p in interval k of plant p (kW)
$QF_{n_p,k}^H$ the amount of heat exchanged between hot oil and cold utility n_p in interval k of plant p (kW)
$QHU_k^{q,p}$ the total heat-exchange rate between all heating utilities in plant q and all cold streams in interval k of plant p (kW)

$QHU_k^{p,q}$ the total heat-exchange rate between all heating utilities in plant p and all cold streams in interval k of plant q (kW)
$QT_{p,k}^C$ the amount of heat absorbed by intermediate fluid as a cold stream in interval k of plant p (kW)
$QT_{p,k}^H$ the amount of heat released by intermediate fluid as a hot stream in interval k of plant p (kW)
S_p^U the utility cost saving of plant p (USD/y)
S_p^T the total cost saving realized by plant p after inter-plant heat integration (USD/y)
TC_p^I the total capital cost of all inner-plant matches of plant p in an interplant heat integration scheme (USD/y)
TPC_p^p a fraction of the total capital cost of interplant matches in plant p and each can be denoted as (i_p, F) (USD/y)
TPH_q^p a fraction of the total capital cost of interplant matches in plant q and each is denoted as (F, j_q) (USD/y)
$TRD_{p,q'}$ the capital cost of interplant match $(i_p, j_{q'})$ shared by plant p (USD/y)
$TRD_{q,p}$ the capitasssssl cost of interplant match (i_q, j_p) shared by plant p (USD/y)
$\mathbf{x}_p$ the strategy vector of plant p
$\overline{Z}_p$ the minimum total utility cost of a standalone HEN in plant p (USD/y)
Z_p' the minimum total utility cost of plant p achieved with interplant heat integration (USD/y)
Z_{i_p,j_p} the binary parameter to denote if the corresponding matches between i_p and j_p are present in HEN
$QS_{m_p,k}$ the consumption rate of hot utility m_p of plant p in temperature interval k (kW)
$QW_{n_p,k}$ the consumption rate of cold utility n_p of plant p in temperature interval k (kW)
RF_k the heat residue from hot oil in interval k of plant p (kW)
$R_{i_p,k}$ the heat residue from hot process stream i in interval k of plant p (kW)
$R_{m_p,k}$ the heat residue from hot utility m in interval k of plant p (kW)
$\gamma_{i_p,j_{q'}}^p$ the proportions of capital cost shared by plant p for the heat exchanger facilitating heat export from hot stream i_p to cold stream $j_{q'}$ (dimensionless)
$\gamma_{i_p,j_{q'}}^{q'}$ the proportions of capital cost shared by plant q' for the heat exchanger facilitating heat export from hot stream i_p to cold stream $j_{q'}$ (dimensionless)
γ_{i_q,j_p}^p the proportions of capital cost shared by plant p for the exchanger facilitating heat import from hot stream$i_q$to cold streamj_p (dimensionless)
γ_{i_q,j_p}^q the proportions of capital cost shared by plant q for the exchanger facilitating heat import from hot stream $i_q$to cold stream j_p (dimensionless)
$\lambda_{p,q'}^O$ the fraction of total heat exported by plant p that is received by plant q'
$\lambda_{q,p}^I$ the fraction of total heat imported by plant p that is released by plant q
$\gamma F_{j_q}^p$ the fraction of the total capital cost of match (F, j_q) in plant q $(q \in PH)$ that is shared by plant p $(p \in PC)$
$\gamma F_{i_q}^p$ the fraction of the total capital cost of match (i_q, F) in plant q $(q \in PC)$ that is shared by plant p $(p \in PH)$

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