Chemical Engineering Research and Design 174 (2021) 137–157

Contents lists available at ScienceDirect

Chemical Engineering Research and Design

journal homepage: www.elsevier.com/locate/cherd

Practicable total-site heat integration plan for retrofitting multiple heat exchanger networks

Fang-Yi Lo, Ching-Hsiang Hsu, Chuei-Tin Chang[*]

Department of Chemical Engineering, National Cheng Kung University, Tainan, 70101, Taiwan, ROC

ARTICLE INFO

Article history:
Received 7 April 2021
Received in revised form 29 July 2021
Accepted 30 July 2021
Available online 9 August 2021

Keywords:
Indirect interplant heat integration
Retrofit design
Mathematical programming model
Cooperative game
Core
Shapley value

ABSTRACT

Total-site heat integration (TSHI) is one of effective ways to improve energy efficiency for the process industry. Generally, the existing chemical plants in an industrial park were rarely constructed simultaneously and it is highly possible that each has already been equipped with an optimal heat exchanger network (HEN) at the time when its construction is completed. To further improve the overall performance of the entire site, a novel retrofit design strategy of these HENs is developed in this study using intermediate fluid(s). A two-stage procedure has been adopted to produce a practicable retrofitting plan. Three different revamp strategies are proposed in this work, while any of them can be applied to generate the cost-optimal designs first via indirect interplant heat exchanges. The benefit allocation schemes are then determined according to core and Shapley values in the second stage. The feasibility and effectiveness of proposed methodology are demonstrated in detail with a three-plant example. It is also plain to observe from the corresponding results that the applicability of the aforementioned design strategy is practicable in realistic environment.

1. Introduction

Efficient energy conservation has recently become an important issue in process design due to the growing concerns about global warming and environmental pollution. The heat exchanger network (HEN) is usually configured within a single chemical plant for the purpose of maximizing heat recovery. During the last few years, a number of studies have been carried out to facilitate grassroots designs for total-site heat integration (TSHI), e.g., Klemes (2013), Wang et al. (2015), Song et al. (2017) and Hong et al. (2019). However, the existing process plants on an industrial park were generally built to satisfy individual market demands which arose during different periods in the past and each plant has already been equipped with a dedicated HEN by the time when its construction was completed. Also, the financial plan for allocating the total cost saving achieved via multi-HEN retrofit has never been studied before. Therefore, this research is aimed to address practical issues concerning realizable retrofit projects for TSHI.

As mentioned above, the HEN has traditionally been installed for maximum heat recovery in a single plant, i.e., Linnhoff and Hindmarsh (1983), Gundersen and Grossmann (1990). The available model-based HEN synthesis strategies may be classified into two types, i.e., the sequential and simultaneous methods. In the former case, the HEN design is produced in three consecutive steps. A linear program (LP) built on the basis of transshipment model (Bradley et al., 1977) is first formulated for calculating the minimum total utility cost. In the second step, a mixed-integer linear programing (MILP) model with the embedded constraint of minimum utility cost obtained in the first step is adopted to determine the minimum number of matches and the corresponding heat duties. The detailed procedures of above two steps were documented in Papoulias and Grossmann (1983). In the final step of the sequential approach, the optimal HEN structure is synthesized by solving a nonlinear programing (NLP) model (Floudas et al., 1986). On the other hand, a superstructure-based mixed-integer nonlinear programing (MINLP) model is adopted in the latter case for identifying the cost-optimal HEN design in one step (Yee and Grossmann, 1990). In general, the simultaneous approach yields a better solution since the total annual cost (TAC) is directly used as the objective function of the aforementioned MINLP model (Liu et al., 2018). For this reason, the simultaneous approach has been adopted as the basis for HEN retrofit in the present work.

---

[*] *Corresponding author.*
E-mail address: ctchang@mail.ncku.edu.tw (C.-T. Chang).

https://doi.org/10.1016/j.cherd.2021.07.032
0263-8762/

It has been well established that the total utility cost of more than one standalone chemical plant can usually be reduced by allowing heat transfers across plant boundaries, e.g., see Bagajewicz and Rodera (2002), Kralj (2008) and Liew et al. (2014, 2017). In general, TSHI can be realized through heat exchanges either between two process streams directly or between a process stream and an intermediate fluid indirectly. A superstructure-based MINLP model for implementing the direct interplant heat integration was proposed in Jin et al. (2018). In contrast to the direct heat exchange method, the indirect alternative is more practicable due to the facts that the latter are more controllable and, also, easier to facilitate the long-distance transport between each pair of standalone plants (Wang and Feng, 2016). Laukkanen et al. (2012) presented a simultaneous approach for TSHI using both direct and indirect interplant heat exchanges. Chang et al. (2015, 2017) adopted the MINLP model to accomplish TSHI using intermediate fluid circuit(s). Based on the above discussions, it is clear that the indirect interplant heat integration is in general more practicable. Therefore, in this study, the heat exchanges between plants are assumed to be always facilitated with an intermediate fluid. It should also be noted that, in recent years, there have been many related studies concerning this issue. Following is a brief survey. Tao et al. (2020) proposed a synthesis method to improve the flexibility of interplant heat exchanger networks via intermediate circles. In particular, the flexibility of the overall system was facilitated by introducing additional utility exchangers not only into inner-plant HENs but also on intermediate fluid circles. Liu et al. (2020) constructed a multi-objective optimization framework for integrating the interplant heat exchanger networks and the shared utility system. In this study, multiple HENs were connected by allowing process streams to produce steam and using this steam as an intermediate fluid for cross-plant heat exchange. Tian et al. (2020a) addressed an indirect TSHI problem using a two-layer simultaneous algorithm. Specifically, the outer layer of the algorithm utilized a differential evolution strategy to compute the temperatures of the intermediate fluid, while the inner layer used a deterministic method to obtain its heat capacity flowrate and the HEN configuration. Finally, Boldyryev et al. (2021) also proposed a Pinch-based approach to TSHI with intermediate utilities.

Although TSHI may result in substantial utility savings, the published studies mostly focused on minimization of total annual cost. Consequently, the resulting integration scheme might not always be acceptable to all participating plants since the total benefit is often not distributed reasonably. To resolve this issue, Cheng et al. (2014) proposed a *sequential* optimization procedure to generate a direct heat integration scheme based on non-cooperative game theory. Chang et al. (2018) further modified the Nash-equilibrium constrained optimization method to synthesize indirect multi-plant HENs. However, it has been well-established that applying the aforementioned sequential design strategies may not reach a true optimum, and it should also be noted that the assumption of non-cooperative game may not be appropriate for a TSHI project.

Hiete et al. (2012) used the thermal pinch analysis for interplant heat integration and treated the aforementioned benefit sharing issue as a cooperative game. Tan et al. (2016) presented a LP cooperative game model for optimal distribution of savings in eco-industrial parks. Jin et al. (2018) developed a model-based two-stage procedure to generate a practical cost-sharing plan in the spirit of a cooperative game. The minimum TAC of every multi-plant HEN was first determined with MINLP model, and the core, the Shapley values and the risk-based Shapley values of a cooperative game were then calculated to settle the cost distribution issues. Tian et al. (2020b) in a later study extended the idea of the risk-based Shapley value for allocating benefits in a two-plant heat integration scheme (which incorporated redundant units for safety reason), while Wang et al. (2020) modified the same concept for indirect TSHI. Notice also that the concepts of the core solution have already been detailed in Fernandez et al. (2002), the Shapley value has been first outlined in Shapley (1953) and the risk-based Shapley value has been described by Grabisch and Xie (2007).

Although the benefit allocation issue can be more appropriately addressed on the basis of cooperative game, the studies mentioned above focused on the grassroots designs only. Ciric and Floudas (1989, 1990) proposed a model-based method for determining the optimum retrofit design of existing HEN configuration in s single plant. Yee and Grossmann (1991) proposed a superstructure-based MINLP model to handle the retrofit design of single-plant HEN. Sorsak and Kravanja (2004) described a MINLP model for the retrofit of single-plant HENs that comprise different exchanger types. Ponce-Ortega et al. (2008) also presented a mathematical model based on superstructure to produce the redesigned HEN that considers the plant layout. Smith et al. (2010) applied the network pinch approach in HEN retrofit design with temperature-dependent thermal properties. From the above discussion, it can be observed that reliable single-plant HEN retrofit methods have already been matured in the past. On the other hand, the model-based multi-plant HEN retrofit method via direct interplant heat integration and the corresponding benefit allocation scheme have also been developed recently by Lo et al. (2020). However, it should be noted that the indirect counterpart of this study is still missing in the literature. Therefore, there are needs to develop retrofit strategies for TSHI via an intermediate fluid, and also to devise practicable allocation plan to distribute the total cost saving.

2. Superstructures for multi-plant HENs

As an example, the superstructures used for two-plant HEN retrofit design via indirect interplant integration are shown in Figs. 1a and 1b. Each plant can be regarded as either a heat source or a heat sink, and a superstructure should be built for each of these two scenarios. Fig. 1a shows the superstructure in which plant P1 takes the source role and P2 the sink, while Fig. 1b shows the counterpart when the source/sink roles are reversed.

In Fig. 1a, CM_{P1} represents the intermediate fluid which is treated as a cold stream for receiving heat from the hot process stream(s) in P1, whereas HM_{P2} denotes the intermediate fluid which is viewed as hot stream for releasing heat to the cold process stream(s) in P2. Plant P1 is equipped with two hot utilities (HP_{P1} and HO_{P1}), one cold utility (CW_{P1}), one hot process stream ($H1_{P1}$), and one cold process stream ($C1_{P1}$). On the other hand, P2 has two hot utilities (HP_{P2} and HO_{P2}), one cold utility (CW_{P2}), one hot process stream ($H1_{P2}$), and one cold process stream ($C1_{P2}$). The matches between process and intermediate streams in both superstructures are gray-colored and those between hot and cold process streams within a plant are represented with uncolored circles. Notice that the number of stages in superstructure for plant P1 should be two, and each stage contains all the possible inner-plant matches between hot stream ($H1_{P1}$) and cold streams ($C1_{P1}$ and CM_{P1}). On the other hand, the number of stages in superstructure of plant P2 should also be two, and each stage contains all the possible inner-plant matches between hot stream ($H1_{P2}$ and HM_{P2}) and cold streams ($C1_{P2}$). In this study, the total number of stages of the overall superstructure (*NOK*) is set to be the larger one of the above two numbers and, thus, should also be two. In addition, notice that an extra bypass is placed on each process stream in every stage and also at the end of stream in order to ensure retrofit flexibility. Finally, notice that the superstructure in Fig. 1b can be constructed in the same way as mentioned above except the roles of P1 (which is now a heat sink) and P2 (which is now a heat source) should be reversed.

The main body of model formulations can be generated based on the proposed superstructures, while additional retrofit constraints are imposed in this work to facilitate realization of specific retrofit strategies. Since the former formulations have

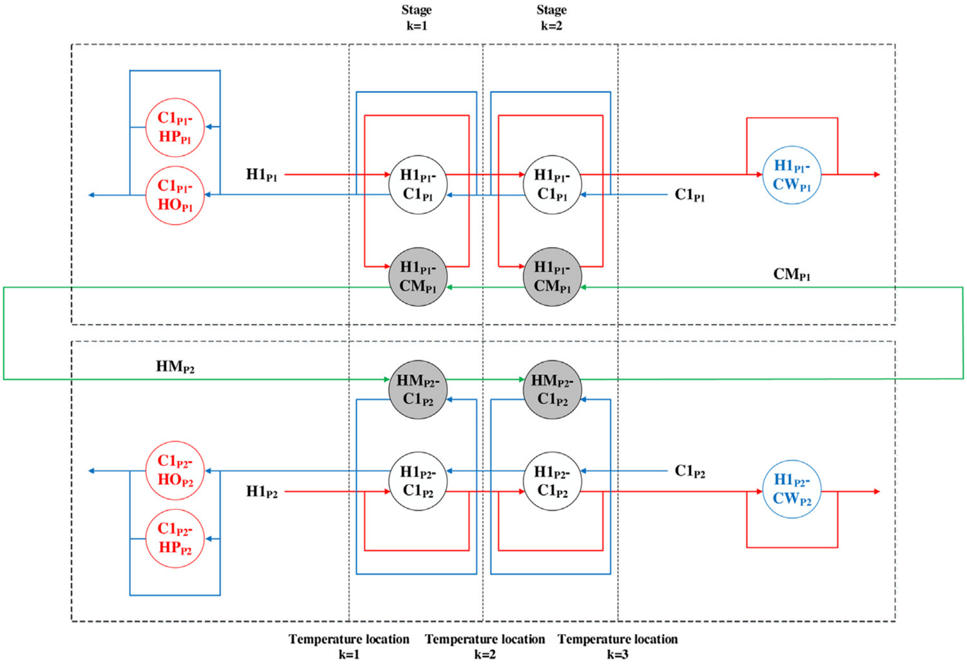

Fig. 1a – Superstructure for two-plant HEN retrofit design via indirect interplant heat integration (P1: heat source; P2: heat sink).

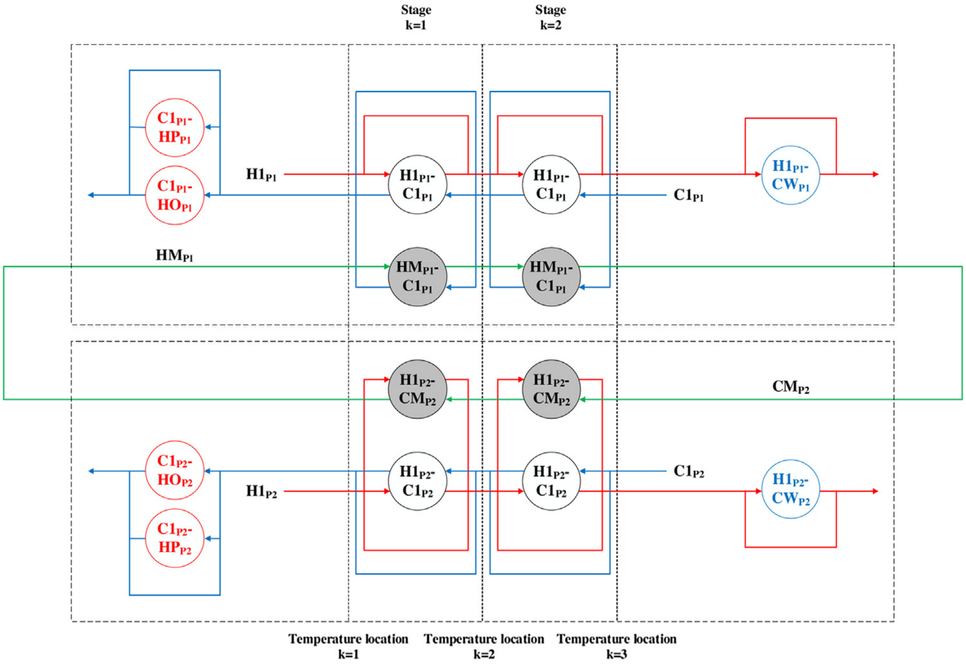

Fig. 1b – Superstructure for two-plant HEN retrofit design via indirect interplant heat integration (P1: heat sink; P2: heat source).

already been published elsewhere (Lo et al., 2020), they are placed in Part A of the Supplementary material to avoid repetition and for the sake of illustration brevity.

3. Revamp strategies

Three retrofit strategies have been devised in this work to facilitate indirect interplant heat integration. Each differs from the others mainly in the reclaimed energy and in the capital investments and re-piping costs of the resulting multi-plant HEN structure. The design guidelines adopt in these three strategies are described below.

- **Strategy I**: Only new inner-plant matches between the process streams and the intermediate streams are allowed to be introduced into the revamp design. The existing exchanger matches located within each plant must be kept unchanged, while the existing cooler and heater matches can only adopt inner-plant utilities. Existing coolers and heaters may be excluded from the revamp design if the corresponding cooling and heating duties are not required. Every inner-plant match between process streams should be housed in its original unit. If a larger heat-transfer area is called for, the heat exchanger can be connected with an extra one in series to fulfil the required heat duty. For the same reason, the existing cooler or heater can also be connected with an extra one in series to fulfil the required heat duty.

- **Strategy II**: Only new inner-plant matches between the process streams and the intermediate streams are allowed to be introduced into the revamp design. The existing exchanger matches located within each plant must be kept unchanged, while the existing cooler and heater matches can only adopt inner-plant utilities. Existing coolers and heaters may be excluded from the revamp design if the corresponding cooling and heating duties are not required. Every inner-plant match between process streams can be housed in either its original heat exchanger or another existing one of the same type within the same plant. If a larger heat-transfer area is called for, the heat exchanger can be connected with an extra one in series to fulfil the required heat duty. The existing cooler or heater can also be connected with an extra one in series to fulfil the required heat duty.
- **Strategy III**: Both the new inner-plant matches between process streams and those between the process and intermediate streams can be introduced into the revamp design, while some of the existing matches may not be utilized. Every existing heat exchanger should be kept within the plant where it is originally located. Existing coolers and heaters may be excluded from the revamp design if the corresponding cooling and heating duties are not required. Any inner-plant match in the revamp design can be housed either in a purchased heat exchanger or an existing one. If a larger heat-transfer area is called for in the latter case, the existing heat exchanger can be connected with an extra one in series to fulfil the required heat duty. The existing cooler or heater can also be connected with an extra one in series to fulfil the required heat duty.

Since a greater financial gain can usually be reaped by solving a MINLP with more relaxed constraints, it can be expected that Strategy III extracts the most benefit, Strategy I the least and Strategy II yields a cost saving that lies between the above two. However, it should be noted that selecting an appropriate strategy actually depends upon additional practical issues also, e.g., budget constraint, safety concerns, spatial limits, operability and controllability, etc. Finally, to facilitate illustration clarity, the unique features of each strategy are summarized in Part B of the Supplementary material.

4. Additional constraints for generating retrofit designs

Other than all the constraints mentioned in Part A of the Supplementary material, it is necessary to incorporate additional ones in the programming model for implementation of aforementioned revamp strategies. These constraints are reported below.

4.1. *Extra constraints needed for applying Strategy I*

In addition to the sets, parameters, and variables outlined in Part A of the Supplementary material, more are introduced below to facilitate illustration of the proposed model formulation.

Sets:

CLM_p	Set of inner-plant matches between cold utilities and hot process streams in the original HEN of plant p
HTM_p	Set of inner-plant matches between hot utilities and cold process streams in the original HEN of plant p
Y^p	Set of existing inner-plant matches in the original HEN of plant p

Parameters:

$A^{EX}_{i_p,j_p,y_{i_p,j_p}}$	Heat-transfer area of existing heat exchanger y_{i_p,j_p} used for housing match (i_p, j_p) in the original HEN of plant p
$A^{EX}_{i_p,n_p}$	Heat-transfer area of existing cooler used for housing match (i_p, n_p) in the original HEN of plant p
$A^{EX}_{m_p,j_p}$	Heat-transfer area of existing heater used for housing match (m_p, j_p) in the original HEN of plant p
$LRRC_{j_p,k}$	Lower bound for bypass flow fraction of cold stream j_p at stage k
$LRRCU_{j_p}$	Lower bound for bypass flow fraction of cold stream j_p at the end of stream
$LRRH_{i_p,k}$	Lower bound for bypass flow fraction of hot stream i_p at stage k
$LRRHU_{i_p}$	Lower bound for bypass flow fraction of hot stream i_p at the end of stream
LX_{i_p,j_p}	Lower bound for heat-transfer area of the augmented unit for housing match (i_p, j_p) in the multi-plant HEN
$LX_{i_p,n_{q'}}$	Lower bound for heat-transfer area of the augmented unit for housing cooler match $\left(i_p, n_{q'}\right)$ in the multi-plant HEN
LX_{m_q,j_p}	Lower bound for heat-transfer area of the augmented unit for housing heater match (m_q, j_p) in the multi-plant HEN
N_{i_p,j_p}	Total number of existing heat exchangers used for housing match (i_p, j_p) in the original HEN of plant p
Λ_{i_p,j_p}	A large enough constant which is not smaller than the largest heat-transfer area of the augmented unit for housing match (i_p, j_p) in the multi-plant HEN

$\Lambda_{i_p,n_{q'}}$	A large enough constant which is not smaller than the largest heat-transfer area of the augmented unit for housing cooler match $\left(i_p, n_{q'}\right)$ in the multi-plant HEN
Λ_{m_q,j_p}	A large enough constant which is not smaller than the largest heat-transfer area of the augmented unit for housing heater match (m_q, j_p) in the multi-plant HEN

Variables:

$e^{y_{i_p,j_p}}_{i_p,j_p,k}$	Binary variable used for determining whether or not the existing heat exchanger y_{i_p,j_p} adopted for housing match (i_p, j_p) in the original HEN of plant p can be used to house the same match at stage k of the multi-plant HEN
$rz_{i_p,k}$	Binary variable used for determining whether or not hot stream i_p requires a bypass stream at stage k of the multi-plant HEN
$rz_{j_p,k}$	Binary variable used for determining whether or not cold stream j_p requires a bypass stream at stage k of the multi-plant HEN
$rzcu_{j_p}$	Binary variable used for determining whether or not cold stream j_p requires a bypass stream for its heater in the multi-plant HEN
$rzhu_{i_p}$	Binary variable used for determining whether or not hot stream i_p requires a bypass stream for its cooler in the multi-plant HEN
$X^{y_{i_p,j_p}}_{i_p,j_p,k}$	Heat-transfer area of the augmented unit of the existing heat exchanger y_{i_p,j_p} adopted for housing match (i_p, j_p) at stage k of the multi-plant HEN
$X_{i_p,n_{q'}}$	Heat-transfer area of the augmented unit of the existing cooler for housing match $\left(i_p, n_{q'}\right)$ in the multi-plant HEN
X_{m_q,j_p}	Heat-transfer area of the augmented unit of the existing heater for housing match (m_q, j_p) in the multi-plant HEN
$\sigma^{y_{i_p,j_p}}_{i_p,j_p,k}$	Binary variable used for determining whether or not the existing heat exchanger y_{i_p,j_p} adopted for housing match (i_p, j_p) in the original HEN of plant p can be used to house the same match at stage of the multi-plant HEN by enlarging its heat-transfer area according to Fig. 2
$\sigma_{i_p,n_{q'}}$	Binary variable used for determining whether or not the existing cooler in the original HEN of plant p can be used to house match $\left(i_p, n_{q'}\right)$ in the multi-plant HEN by enlarging its heat-transfer area according to Fig. 2
σ_{m_q,j_p}	Binary variable used for determining whether or not the existing heater in the original HEN of plant p can be used to house match (m_q, j_p) in the multi-plant HEN by enlarging its heat-transfer area according to Fig. 2

Firstly, to constrain the number of existing heat exchangers, coolers and heaters in the multi-plant HEN, the following inequalities should be incorporated:

$$\sum_{k \in ST} \xi_{i_p,j_p,k} \leq N_{i_p,j_p};\ i_p \in H^p;\ j_p \in C^p;\ (i_p, j_p) \in Y_p \tag{1}$$

$$\xi^{CU}_{i_p,n_p} \leq 1;\ i_p \in H^p;\ n_p \in CU^p;\ (i_p, n_p) \in CLM_p \tag{2}$$

$$\xi^{HU}_{m_p,j_p} \leq 1;\ m_p \in HU^p;\ j_p \in C^p;\ (m_p, j_p) \in HTM_p \tag{3}$$

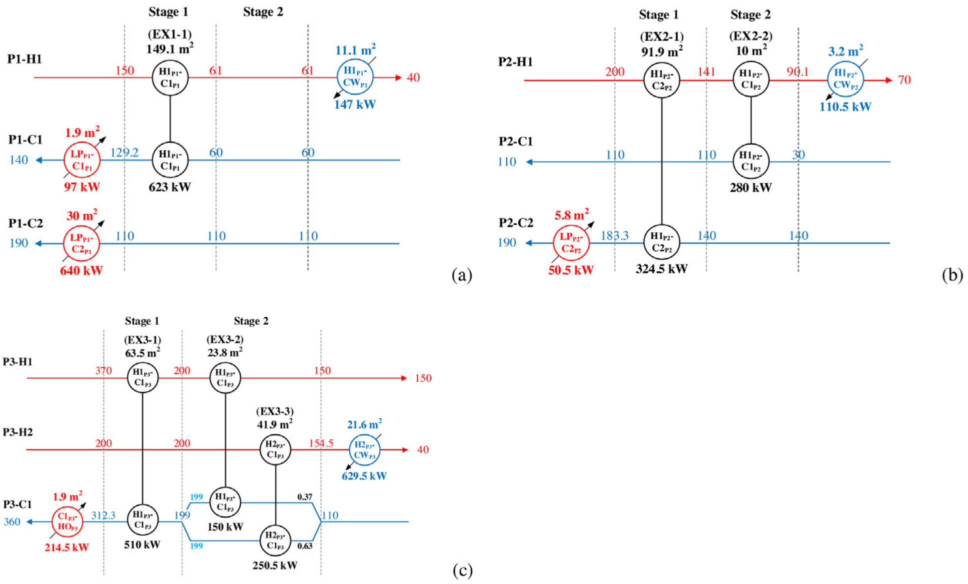

Fig. 2 – Single-plant optimal HENs of (a) P1, (b) P2 and (c) P3.

where, $p = 1, 2, \cdots, P$. Note that the binary variables $\xi_{i_p,j_p,k}$, $\xi^{CU}_{i_p,n_p}$ and $\xi^{HU}_{m_p,j_p}$ should be set to zero if they are not associated with the existing matches, i.e.,

$$\xi_{i_p,j_p,k} = 0;\ i_p \in H^p;\ j_p \in C^p;\ (i_p, j_p) \notin Y_p;\ k \in ST \tag{4}$$

$$\xi^{CU}_{i_p,n_p} = 0;\ i_p \in H^p;\ n_p \in CU^p;\ (i_p, n_p) \notin CLM_p \tag{5}$$

$$\xi^{HU}_{m_p,j_p} = 0;\ m_p \in HU^p;\ j_p \in C^p;\ (m_p, j_p) \notin HTM_p \tag{6}$$

Exactly one existing heat exchanger (which is originally used to house an existing match in single-plant HEN) can be used to house the *same* match in a distinct stage in the multi-plant HEN, i.e.,

$$\sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}} e^{y_{i_p,j_p}}_{i_p,j_p,k} - \xi_{i_p,j_p,k} = 0;\ i_p \in H^p;\ j_p \in C^p;\ (i_p, j_p) \in Y_p;\ k \in ST \tag{7}$$

$$\sum_{k \in ST} e^{y_{i_p,j_p}}_{i_p,j_p,k} = 1;\ i_p \in H^p;\ j_p \in C^p;\ (i_p, j_p) \in Y_p;\ y_{i_p,j_p} = 1, 2, 3, \ldots, N_{i_p,j_p} \tag{8}$$

Note also that if the existing heat exchangers, coolers and heaters can be adopted in the multi-plant HEN, the corresponding heat-transfer areas should be constrained as follows:

$$\begin{gathered} A_{i_p,j_p,k} - A^{EX}_{i_p,j_p,y_{i_p,j_p}} e^{y_{i_p,j_p}}_{i_p,j_p,k} \geq 0; \\ i_p \in H^p;\ j_p \in C^p;\ (i_p, j_p) \in Y_p;\ k \in ST;\ y_{i_p,j_p} = 1, 2, 3, \ldots, N_{i_p,j_p} \end{gathered} \tag{9}$$

$$A_{i_p,n_p} - A^{EX}_{i_p,n_p} \xi^{CU}_{i_p,n_p} \geq 0;\ i_p \in H^p;\ n_p \in CU^p;\ (i_p, n_p) \in CLM_p \tag{10}$$

$$A_{m_p,j_p} - A^{EX}_{m_p,j_p} \xi^{HU}_{m_p,j_p} \geq 0;\ m_p \in HU^p;\ j_p \in C^p;\ (m_q, j_p) \in HTM_p \tag{11}$$

The existing heat exchangers, coolers and heaters can be connected with an extra one in series to fulfil the required heat duty according to Fig. 2. The augmented heat-transfer area ($X^{y_{i_p,j_p}}_{i_p,j_p,k}$, X_{i_p,n_p} and X_{m_p,j_p}) can be expressed as follows:

$$\begin{gathered} X^{y_{i_p,j_p}}_{i_p,j_p,k} = \left(A_{i_p,j_p,k} - A^{EX}_{i_p,j_p,y_{i_p,j_p}}\right) e^{y_{i_p,j_p}}_{i_p,j_p,k}; \\ i_p \in H^p;\ j_p \in C^p;\ (i_p, j_p) \in Y_p;\ k \in ST;\ y_{i_p,j_p} = 1, 2, 3, \ldots, N_{i_p,j_p} \end{gathered} \tag{12}$$

$$X_{i_p,n_p} = \left(A_{i_p,n_p} - A^{EX}_{i_p,n_p}\right) \xi^{CU}_{i_p,n_p};\ i_p \in H^p;\ n_p \in CU^p;\ (i_p, n_p) \in CLM_p \tag{13}$$

$$X_{m_p,j_p} = \left(A_{m_p,j_p} - A^{EX}_{m_p,j_p}\right) \xi^{HU}_{m_p,j_p};\ m_p \in HU^p;\ j_p \in C^p;\ (m_p, j_p) \in HTM_p \tag{14}$$

To facilitate calculation of the capital cost of augmented unit for each existing match in the multi-plant HEN, the following logic constraints must be imposed:

$$\begin{gathered} X^{y_{i_p,j_p}}_{i_p,j_p,k} - \Lambda_{i_p,j_p} \sigma^{y_{i_p,j_p}}_{i_p,j_p,k} \leq 0; \\ i_p \in H^p;\ j_p \in C^p;\ (i_p, j_p) \in Y^p;\ k \in ST;\ y_{i_p,j_p} = 1, 2, 3, \ldots, N_{i_p,j_p} \end{gathered} \tag{15}$$

$$X_{i_p,n_p} - \Lambda_{i_p,n_p} \sigma_{i_p,n_p} \leq 0;\ i_p \in H^p;\ n_p \in CU^p;\ (i_p, n_p) \in CLM_p \tag{16}$$

$$X_{m_p,j_p} - \Lambda_{m_p,j_p} \sigma_{m_p,j_p} \leq 0;\ m_p \in HU^p;\ j_p \in C^p;\ (m_p, j_p) \in HTM_p \tag{17}$$

Similarly, to facilitate calculation of the total capital cost of bypasses in the multi-plant HEN, the following logic constraints must also be imposed:

$$rrh_{i_p,k} - rz_{i_p,k} \leq 0;\ i_p \in H^p;\ k \in ST \tag{18}$$

$$rrh_{j_p,k} - rz_{j_p,k} \leq 0;\ j_p \in C^p;\ k \in ST \tag{19}$$

$$rrhu_{i_p} - rzhu_{i_p} \leq 0;\ i_p \in H^p \tag{20}$$

$$rrcu_{j_p} - rzcu_{j_p} \leq 0;\ j_p \in C^p \tag{21}$$

Also, to avoid impractically small heat-transfer area of the augmented units, the following constraints should be incorporated:

$$X^{y_{i_p,j_p}}_{i_p,j_p,k} - LX_{i_p,j_p}\sigma^{y_{i_p,j_p}}_{i_p,j_{p,k}} \geq 0;$$
$$i_p \in H^p;\ j_p \in C^p;\ (i_p, j_p) \in Y^p;\ k \in ST;\ y_{i_p,j_p} = 1, 2, 3, \ldots, N_{i_p,j_p} \tag{22}$$

$$X_{i_p,n_p} - LX_{i_p,n_p}\sigma_{i_p,n_p} \geq 0;\ i_p \in H^p;\ n_p \in CU^p;\ (i_p, n_p) \in CLM_p \tag{23}$$

$$X_{m_p,j_p} - LX_{m_p,j_p}\sigma_{m_p,j_p} \geq 0;\ m_p \in HU^p;\ j_p \in C^p;\ (m_p, j_p) \in HTM_p \tag{24}$$

Similarly, every impractically small bypass flow fraction should also be prohibited as follows:

$$rrh_{i_p,k} - LRRH_{i_p,k}rz_{i_p,k} \geq 0;\ i_p \in H^p;\ k \in ST \tag{25}$$

$$rrc_{j_p,k} - LRRC_{j_p,k}rz_{j_p,k} \geq 0;\ j_p \in C^p;\ k \in ST \tag{26}$$

$$rrhu_{i_p} - LRRHU_{i_p}rzhu_{i_p} \geq 0;\ i_p \in H^p \tag{27}$$

$$rrcu_{j_p} - LRRCU_{j_p}rzcu_{j_p} \geq 0;\ j_p \in C^p \tag{28}$$

4.2. *Extra constraints needed for applying Strategy II*

In addition to the sets, parameters, and variables defined in Part A of the Supplementary material and Subsection 4.1, more are given below for illustration convenience.

Sets:

$CL^{w_p}_p$	Set of existing cooler matches (i_p, n_p) of type w_p in plant p
$HT^{l_p}_p$	Set of existing heater matches (m_p, j_p) of type l_p in plant p

Parameters:

L_p	Total number of heater types in plant p
N_p	Total number of existing heat exchangers in the original HEN in plant p
$N^{w_p}_{CL}$	Total number of existing coolers of type w_p in the original HEN of plant p
$N^{l_p}_{HT}$	Total number of existing heaters of type l_p in the original HEN of plant p
W_p	Total number of cooler types in plant p

Variables:

$e^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k}$	Binary variable used for determining whether or not the existing heat exchanger y_{i_p,j_p} adopted for housing match (i_p, j_p) in the original HEN of plant p can be used to house another match $\left(\tilde{i}_p, \tilde{j}_p\right)$ at stage k of the multi-plant HEN
$e^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p}$	Binary variable used for determining whether or not an existing cooler of type w_p for housing match (i_p, n_p) in the original HEN of plant p can be used to house another cooler match $\left(\tilde{i}_p, \tilde{n}_p\right)$ in the multi-plant HEN
$e^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p}$	Binary variable used for determining whether or not an existing heater of type l_p for housing match (m_p, j_p) in the original HEN of plant p can be used to house another heater match $\left(\tilde{m}_p, \tilde{j}_p\right)$ in the multi-plant HEN
$X^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k}$	Heat-transfer area of the augmented unit of the existing heat exchanger y_{i_p,j_p} of match (i_p, j_p) in the original single-plant HEN for housing another match $\left(\tilde{i}_p, \tilde{j}_p\right)$ at stage k of the multi-plant HEN
$X^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p}$	Heat-transfer area of the augmented unit of the existing cooler of type w_p, i.e., $(i_p, n_p) \in CL^{w_p}_p$, in the original HEN of plant p for housing another cooler match $\left(\tilde{i}_p, \tilde{n}_p\right)$ in the multi-plant HEN
$X^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p}$	Heat-transfer area of the augmented unit of the existing heater of type l_p, i.e., $(m_p, j_p) \in HT^{l_p}_p$, in the original HEN of plant p for housing another heater match $\left(\tilde{m}_p, \tilde{j}_p\right)$ in the multi-plant HEN
$\sigma^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k}$	Binary variable used for determining whether or not the existing heat exchanger y_{i_p,j_p} adopted for housing match (i_p, j_p) in the original HEN of plant p can be used to house another match $\left(\tilde{i}_p, \tilde{j}_p\right)$ at stage k of the multi-plant HEN by enlarging its heat-transfer area according to Fig. 2

$\sigma_{\tilde{i}_p,\tilde{n}_p}^{i_p,n_p,w_p}$	Binary variable used for determining whether or not the existing cooler of type w_p, i.e., $(i_p, n_p) \in CL_p^{w_p}$, in the original HEN of plant p can be used to house another cooler match $\left(\tilde{i}_p, \tilde{n}_p\right)$ in the multi-plant HEN by enlarging its heat-transfer area according to Fig. 2
$\sigma_{\tilde{m}_p,\tilde{j}_p}^{m_p,j_p,l_p}$	Binary variable used for determining whether or not the existing heater of type l_p, i.e., $(m_p, j_p) \in HT_p^{l_p}$, in the original HEN of plant p can be used to house another heater match $\left(\tilde{m}_p, \tilde{j}_p\right)$ in the multi-plant HEN by enlarging its heat-transfer area according to Fig. 2

First of all, to constrain the total number of existing heat exchangers in the multi-plant HEN, the following inequalities should be imposed:

$$\sum_{(i_p,j_p) \in Y_p} \sum_{k \in ST} \xi_{i_p,j_p,k} \le N_p \tag{29}$$

where, $p = 1, 2, \ldots, P$. On the other hand, to constrain the total number of existing coolers and heaters of the same types, the following inequalities should be used:

$$\sum_{(i_p,n_p) \in CL_p^{w_p}} \xi_{i_p,n_p}^{CU} \le N_{CL}^{w_p};\ w_p = 1, 2, \cdots, W_p \tag{30}$$

$$\sum_{(m_p,j_p) \in HT_p^{l_p}} \xi_{m_p,j_p}^{HU} \le N_{HT}^{l_p};\ l_p = 1, 2, \cdots, L_p \tag{31}$$

In addition, Eqs. (4)–(6) are still valid in the present case. Since every existing match can be housed in either its original heat exchanger or another existing one of the same type within the same plant, the corresponding constraints should be expressed as:

$$\begin{gathered}\sum_{(i_p,j_p) \in Y^p} \sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}} e_{\tilde{i}_p,\tilde{j}_p,k}^{i_p,j_p,y_{i_p,j_p}} - \xi_{\tilde{i}_p,\tilde{j}_p,k} = 0; \\ \tilde{i}_p \in H^p;\ \tilde{j}_p \in C^p;\ \left(\tilde{i}_p, \tilde{j}_p\right) \in Y^p;\ k \in ST\end{gathered} \tag{32}$$

$$\begin{gathered}\sum_{(i_p,n_p) \in CL_p^{w_p}} e_{\tilde{i}_p,\tilde{n}_p}^{i_p,n_p,w_p} - \xi_{\tilde{i}_p,\tilde{n}_p}^{CU} = 0; \\ \tilde{i}_p \in H^p;\ \tilde{n}_p \in CU^p;\ \left(\tilde{i}_p, \tilde{n}_p\right) \in CL_p^{w_p};\ w_p = 1, 2, \cdots, W_p\end{gathered} \tag{33}$$

$$\begin{gathered}\sum_{(m_p,j_p) \in HT_p^{l_p}} e_{\tilde{m}_p,\tilde{j}_p}^{m_p,j_p,l_p} - \xi_{\tilde{m}_p,\tilde{j}_p}^{HU} = 0; \\ \tilde{m}_p \in HU^p;\ \tilde{j}_p \in C^p;\ \left(\tilde{m}_p, \tilde{j}_p\right) \in HT_p^{l_p};\ l_p = 1, 2, \cdots, L_p\end{gathered} \tag{34}$$

Furthermore, each existing heat exchanger should be used to house exactly one match in the multi-plant HEN, i.e.,

$$\begin{gathered}\sum_{\left(\tilde{i}_p,\tilde{j}_p\right) \in Y^p} \sum_{k \in ST} e_{\tilde{i}_p,\tilde{j}_p,k}^{i_p,j_p,y_{i_p,j_p}} = 1; \\ i_p \in H^p;\ j_p \in C^p;\ (i_p, j_p) \in Y^p;\ y_{i_p,j_p} = 1, 2, \cdots, N_{i_p,j_p}\end{gathered} \tag{35}$$

Also, if the existing units can be adopted in the multi-plant HEN, the corresponding heat-transfer areas should be constrained as follows:

$$\begin{gathered}A_{\tilde{i}_p,\tilde{j}_p,k} - A_{i_p,j_p,y_{i_p,j_p}}^{EX} e_{\tilde{i}_p,\tilde{j}_p,k}^{i_p,j_p,y_{i_p,j_p}} \ge 0; \\ i_p, \tilde{i}_p \in H^p;\ j_p, \tilde{j}_p \in C^p;\ (i_p, j_p), \left(\tilde{i}_p, \tilde{j}_p\right) \in Y^p;\ k \in ST;\ y_{i_p,j_p} = 1, 2, \cdots, N_{i_p,j_p}\end{gathered} \tag{36}$$

$$\begin{gathered}A_{\tilde{i}_p,\tilde{n}_p} - A_{i_p,n_p}^{EX} e_{\tilde{i}_p,\tilde{n}_p}^{i_p,n_p,w_p} \ge 0;\ i_p, \tilde{i}_p \in H^p;\ n_p, \tilde{n}_p \in CU^p; \\ (i_p, n_p), \left(\tilde{i}_p, \tilde{n}_p\right) \in CL_p^{w_p};\ w_p = 1, 2, \cdots, W_p\end{gathered} \tag{37}$$

$$A_{\tilde{m}_p,\tilde{j}_p} - A^{EX}_{m_p,j_p} e^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} \geq 0;\ m_p, \tilde{m}_p \in HU^p;\ j_p, \tilde{j}_p \in C^p; \\ (m_p, j_p), \left(\tilde{m}_p, \tilde{j}_p\right) \in HT^{l_p}_p;\ l_p = 1, 2, \cdots, L_p \tag{38}$$

Any existing heat exchanger, cooler or and heater can be connected with an extra one in series to fulfil the required heat duty according to Fig. 2. The corresponding augmented heat-transfer area ($X^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k}$, $X^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p}$ and $X^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p}$) can be expressed as follows:

$$X^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} = \left(A_{\tilde{i}_p,\tilde{j}_p,k} - A^{EX}_{y_{i_p,j_p}}\right) e^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k}; \\ i_p, \tilde{i}_p \in H^p;\ j_p, \tilde{j}_p \in C^p;\ (i_p, j_p), \left(\tilde{i}_p, \tilde{j}_p\right) \in Y^p;\ k \in ST;\ y_{i_p,j_p} = 1, 2, \cdots, N_{i_p,j_p} \tag{39}$$

$$X^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} = \left(A_{\tilde{i}_p,\tilde{n}_p} - A^{EX}_{i_p,n_p}\right) e^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p};\ i_p, \tilde{i}_p \in H^p;\ n_p, \tilde{n}_p \in CU^p; \\ (i_p, n_p), \left(\tilde{i}_p, \tilde{n}_p\right) \in CL^{w_p}_p;\ w_p = 1, 2, \cdots, W_p \tag{40}$$

$$X^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} = \left(A_{\tilde{m}_p,\tilde{j}_p} - A^{EX}_{m_p,j_p}\right) e^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p};\ m_p, \tilde{m}_p \in HU^p;\ j_p, \tilde{j}_p \in C^p; \\ (m_p, j_p), \left(\tilde{m}_p, \tilde{j}_p\right) \in HT^{l_p}_p;\ l_p = 1, 2, \cdots, L_p \tag{41}$$

To facilitate calculation of the capital cost of augmented unit for each existing match in the multi-plant HEN, the following logic constraints must be imposed:

$$X^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} - \Lambda_{\tilde{i}_p,\tilde{j}_p} \sigma^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} \leq 0; \\ i_p, \tilde{i}_p \in H^p;\ j_p, \tilde{j}_p \in C^p;\ (i_p, j_p), \left(\tilde{i}_p, \tilde{j}_p\right) \in Y^p;\ k \in ST;\ y_{i_p,j_p} = 1, 2, \cdots, N_{i_p,j_p} \tag{42}$$

$$X^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} - \Lambda_{\tilde{i}_p,\tilde{n}_p} \sigma^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} \leq 0;\ i_p, \tilde{i}_p \in H^p;\ n_p, \tilde{n}_p \in CU^p; \\ (i_p, n_p), \left(\tilde{i}_p, \tilde{n}_p\right) \in CL^{w_p}_p;\ w_p = 1, 2, \cdots, W_p \tag{43}$$

$$X^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} - \Lambda_{\tilde{m}_p,\tilde{j}_p} \sigma^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} \leq 0;\ m_p, \tilde{m}_p \in HU^p;\ j_p, \tilde{j}_p \in C^p; \\ (m_p, j_p), \left(\tilde{m}_p, \tilde{j}_p\right) \in HT^{l_p}_p;\ l_p = 1, 2, \cdots, L_p \tag{44}$$

Also, to avoid impractically small heat-transfer area of the augmented units, the following constraints should be incorporated:

$$X^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} - LX_{\tilde{i}_p,\tilde{j}_p} \sigma^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} \geq 0; \\ i_p, \tilde{i}_p \in H^p;\ j_p, \tilde{j}_p \in C^p;\ (i_p, j_p), \left(\tilde{i}_p, \tilde{j}_p\right) \in Y^p;\ k \in ST;\ y_{i_p,j_p} = 1, 2, \cdots, N_{i_p,j_p} \tag{45}$$

$$X^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} - LX_{\tilde{i}_p,\tilde{n}_p} \sigma^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} \geq 0;\ i_p, \tilde{i}_p \in H^p;\ n_p, \tilde{n}_p \in CU^p; \\ (i_p, n_p), \left(\tilde{i}_p, \tilde{n}_p\right) \in CL^{w_p}_p;\ w_p = 1, 2, \cdots, W_p \tag{46}$$

$$X^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} - LX_{\tilde{m}_p,\tilde{j}_p} \sigma^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} \geq 0;\ m_p, \tilde{m}_p \in HU^p;\ j_p, \tilde{j}_p \in C^p; \\ (m_p, j_p), \left(\tilde{m}_p, \tilde{j}_p\right) \in HT^{l_p}_p;\ l_p = 1, 2, \cdots, L_p \tag{47}$$

Finally, notice that Eqs. (18)–(21) and (25)–(28) should also be included in the present application.

4.3. *Extra constraints needed for applying Strategy III*

In addition to the sets, parameters, and variables defined in Part A of the Supplementary material and Subsections 4.1 and 4.2, more are presented in the sequel for illustration clarity.

Sets:

Z_p	Set of all possible inner-plant matches between hot stream $\tilde{i}_p$ and cold stream $\tilde{j}_p$ in plant p
$ZCL_p^{w_p}$	Set of all possible inner-plant cooler matches between hot stream $\tilde{i}_p$ and cold utility $\tilde{n}_p$ which can be housed in coolers of type w_p
$ZHT_p^{l_p}$	Set of all possible inner-plant heater matches between hot utility $\tilde{m}_p$ and cold stream $\tilde{j}_p$ which can be housed in heaters of type l_p

Variables:

$u_{\tilde{i}_p,\tilde{j}_p,k}$	Binary variable used for determining whether or not a new heat exchanger should be purchased to house inner-plant match $(\tilde{i}_p, \tilde{j}_p)$ at stage k of the multi-plant HEN
$u^{CU,w_p}_{\tilde{i}_p,\tilde{n}_p}$	Binary variable used for determining whether or not a new cooler of type w_p should be purchased to house inner-plant cooler match $(\tilde{i}_p, \tilde{n}_p)$ in the multi-plant HEN
$u^{HU,l_p}_{\tilde{m}_p,\tilde{j}_p}$	Binary variable used for determining whether or not a new heater of type l_p should be purchased to house inner-plant heater match $(\tilde{m}_p, \tilde{j}_p)$ in the multi-plant HEN

In the multi-plant HEN, the total numbers of heat exchangers, coolers and heaters that are housed in existing units should be bounded as follows

$$\sum_{(i_p,j_p)\in Y^p}\sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}}\sum_{\left(\tilde{i}_p,\tilde{j}_p\right)\in Z^p}\sum_{k\in ST} e^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} \le N_p \tag{48}$$

$$\sum_{(i_p,n_p)\in CL_p^{w_p}}\sum_{\left(\tilde{i}_p,\tilde{n}_p\right)\in ZCL_p^{w_p}} e^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} \le N^{w_p}_{CL};\ w_p = 1, 2, \cdots, W_p \tag{49}$$

$$\sum_{(m_p,j_p)\in HT_p^{l_p}}\sum_{\left(\tilde{m}_p,\tilde{j}_p\right)\in ZHT_p^{l_p}} e^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} \le N^{l_p}_{HT};\ l_p = 1, 2, \cdots, L_p \tag{50}$$

For housing the exchanger, cooler and heater matches in multi-plant HEN with either available or new units, the following constraints should be adopted:

$$\begin{gathered}\sum_{(i_p,j_p)\in Y^p}\sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}} e^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} + u_{\tilde{i}_p,\tilde{j}_p,k} - \xi_{\tilde{i}_p,\tilde{j}_p,k} = 0;\\ \tilde{i}_p \in H^p;\ \tilde{j}_p \in C^p;\ \left(\tilde{i}_p,\tilde{j}_p\right) \in Z^p;\ k \in ST\end{gathered} \tag{51}$$

$$\begin{gathered}\sum_{(i_p,n_p)\in CL_p^{w_p}} e^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} + u^{CU,w_p}_{\tilde{i}_p,\tilde{n}_p} - \xi^{CU}_{\tilde{i}_p,\tilde{n}_p} = 0;\\ \tilde{i}_p \in H^p;\ \tilde{n}_p \in CU^p;\ \left(\tilde{i}_p,\tilde{n}_p\right) \in ZCL_p^{w_p};\ w_p = 1, 2, \cdots, W_p\end{gathered} \tag{52}$$

$$\begin{gathered}\sum_{(m_p,j_p)\in HT_p^{l_p}} e^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} + u^{HU,l_p}_{\tilde{m}_p,\tilde{j}_p} - \xi^{HU}_{\tilde{m}_p,\tilde{j}_p} = 0;\\ \tilde{m}_p \in HU^p;\ \tilde{j}_p \in C^p;\ \left(\tilde{m}_p,\tilde{j}_p\right) \in HT_p^{l_p};\ l_p = 1, 2, \cdots, L_p\end{gathered} \tag{53}$$

Every existing heat exchanger should of course be used to house exactly one exchanger match in the multi-plant HEN, i.e.,

$$\sum_{\left(\tilde{i}_p,\tilde{j}_p\right)\in Z^p}\sum_{k\in ST} e^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} = 1;\ (i_p,j_p) \in Y^p;\ y_{i_p,j_p} = 1, 2, \cdots, N_{i_p,j_p} \tag{54}$$

Also, if the existing units can be utilized in the multi-plant HEN, the corresponding heat-transfer areas should be constrained as follows:

$$A_{\tilde{i}_p,\tilde{j}_p,k} - A^{EX}_{i_p,j_p,y_{i_p,j_p}} e^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} \geq 0;\ i_p,\tilde{i}_p \in H^p;\ j_p,\tilde{j}_p \in C^p;$$
$$(i_p,j_p) \in Y^p;\ \left(\tilde{i}_p,\tilde{j}_p\right) \in Z^p;\ k \in ST;\ y_{i_p,j_p} = 1,2,\cdots,N_{i_p,j_p} \tag{55}$$

$$A_{\tilde{i}_p,\tilde{n}_p} - A^{EX}_{i_p,n_p} e^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} \geq 0;\ i_p,\tilde{i}_p \in H^p;\ n_p,\tilde{n}_p \in CU^p;$$
$$(i_p,n_p) \in CL^{w_p}_p;\ \left(\tilde{i}_p,\tilde{n}_p\right) \in ZCL^{w_p}_p;\ w_p = 1,2,\cdots,W_p \tag{56}$$

$$A_{\tilde{m}_p,\tilde{j}_p} - A^{EX}_{m_p,j_p} e^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} \geq 0;\ m_p,\tilde{m}_p \in HU^p;\ j_p,\tilde{j}_p \in C^p;$$
$$(m_p,j_p) \in HT^{l_p}_p;\ \left(\tilde{m}_p,\tilde{j}_p\right) \in ZHT^{l_p}_p;\ l_p = 1,2,\cdots,L_p \tag{57}$$

Each existing exchanger, cooler or heater can be connected with an extra one in series to fulfil the required heat duty according to Fig. 2. The corresponding augmented heat-transfer area ($X^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k}$, $X^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p}$ and $X^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p}$) can be expressed as follows:

$$X^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} = \left(A_{\tilde{i}_p,\tilde{j}_p,k} - A^{EX}_{y_{i_p,j_p}}\right) e^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k};\ i_p,\tilde{i}_p \in H^p;\ j_p,\tilde{j}_p \in C^p;$$
$$(i_p,j_p) \in Y^p;\ \left(\tilde{i}_p,\tilde{j}_p\right) \in Z^p;\ k \in ST;\ y_{i_p,j_p} = 1,2,\cdots,N_{i_p,j_p} \tag{58}$$

$$X^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} = \left(A_{\tilde{i}_p,\tilde{n}_p} - A^{EX}_{i_p,n_p}\right) e^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p};\ i_p,\tilde{i}_p \in H^p;\ n_p,\tilde{n}_p \in CU^p;$$
$$(i_p,n_p) \in CL^{w_p}_p;\ \left(\tilde{i}_p,\tilde{n}_p\right) \in ZCL^{w_p}_p;\ w_p = 1,2,\cdots,W_p \tag{59}$$

$$X^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} = \left(A_{\tilde{m}_p,\tilde{j}_p} - A^{EX}_{m_p,j_p}\right) e^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p};\ m_p,\tilde{m}_p \in HU^p;\ j_p,\tilde{j}_p \in C^p;$$
$$(m_p,j_p) \in HT^{l_p}_p;\ \left(\tilde{m}_p,\tilde{j}_p\right) \in ZHT^{l_p}_p;\ l_p = 1,2,\cdots,L_p \tag{60}$$

The presence (or absence) of an augmented unit can be determined with the binary variables in the following logic constraints:

$$X^{y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} - \Lambda_{\tilde{i}_p,\tilde{j}_p} \sigma^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} \leq 0;\ i_p,\tilde{i}_p \in H^p;\ j_p,\tilde{j}_p \in C^p;$$
$$(i_p,j_p) \in Y^p;\ \left(\tilde{i}_p,\tilde{j}_p\right) \in Z^p;\ k \in ST;\ y_{i_p,j_p} = 1,2,\cdots,N_{i_p,j_p} \tag{61}$$

$$X^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} - \Lambda_{\tilde{i}_p,\tilde{n}_p} \sigma^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} \leq 0;\ i_p,\tilde{i}_p \in H^p;\ n_p,\tilde{n}_p \in CU^p;$$
$$(i_p,n_p) \in CL^{w_p}_p;\ \left(\tilde{i}_p,\tilde{n}_p\right) \in ZCL^{w_p}_p;\ w_p = 1,2,\cdots,W_p \tag{62}$$

$$X^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} - \Lambda_{\tilde{m}_p,\tilde{j}_p} \sigma^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} \leq 0;\ m_p,\tilde{m}_p \in HU^p;\ j_p,\tilde{j}_p \in C^p;$$
$$(m_p,j_p) \in HT^{l_p}_p;\ \left(\tilde{m}_p,\tilde{j}_p\right) \in ZHT^{l_p}_p;\ l_p = 1,2,\cdots,L_p \tag{63}$$

In addition, to avoid using an augmented unit with impractically small heat-transfer area, the following constraints should be incorporated:

$$X^{y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} - LX_{\tilde{i}_p,\tilde{j}_p} \sigma^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} \geq 0;\ i_p,\tilde{i}_p \in H^p;\ j_p,\tilde{j}_p \in C^p;$$
$$(i_p,j_p) \in Y^p;\ \left(\tilde{i}_p,\tilde{j}_p\right) \in Z^p;\ k \in ST;\ y_{i_p,j_p} = 1,2,\cdots,N_{i_p,j_p} \tag{64}$$

$$X^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} - LX_{\tilde{i}_p,\tilde{n}_p} \sigma^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} \geq 0;\ i_p,\tilde{i}_p \in H^p;\ n_p,\tilde{n}_p \in CU^p;$$
$$(i_p,n_p) \in CL^{w_p}_p;\ \left(\tilde{i}_p,\tilde{n}_p\right) \in ZCL^{w_p}_p;\ w_p = 1,2,\cdots,W_p \tag{65}$$

$$X^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} - LX_{\tilde{m}_p,\tilde{j}_p}\,\sigma^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} \geq 0;\ m_p,\tilde{m}_p \in HU^p;\ j_p,\tilde{j}_p \in C^p;$$
$$(m_p,j_p) \in HT_p^{l_p};\ \left(\tilde{m}_p,\tilde{j}_p\right) \in ZHT_p^{l_p};\ l_p = 1,2,\cdots,L_p \tag{66}$$

Finally, notice that Eqs. (18)–(21) and (25)–(28) should also be included in the present application.

5. Objective functions

In this study, the *total annual cost saving* (TACS) achieved by transforming the single-plant HENs into a multi-plant HEN is used as the objective function to be maximized. This function can be expressed as follows:

$$TACS = TUC - TUC' - AF\,(ATCC + NTCC_1 + NTCC_2 + NTCC_3 + NTCC_4 + TIMC) \tag{67}$$

where, TUC denotes the sum of annual utility costs of all single-plant HENs which is regarded as a given constant in the corresponding MINLP models; TUC' denotes the total annual utility cost of the multi-plant HEN after retrofit; AF is the annualization factor which is another given constant; $ATCC$ is the total capital cost of all augmented units; $NTCC_1$ denotes the total capital cost of all new units purchased for inner-plant matches between process and intermediate streams; $NTCC_2$ denotes the total capital cost of all bypasses; $NTCC_3$ denotes the total unit reassignment cost; $NTCC_4$ denotes the total capital cost of all new units purchased for inner-plant matches between hot and cold process streams; $TIMC$ is the total cost of intermediate fluid. Other than the aforementioned two constants (i.e., TUC and AF), the detailed expressions of the remaining cost models are listed in the subsequent subsections, and, for the sake of brevity, all embedded model parameters (or cost coefficients) are first defined below:

Parameters:

BY_{i_p}	Cost of a single bypass on hot stream i_p
BY_{j_p}	Cost of a single bypass on cold stream j_p
CA_{hm_p,j_p}	Variable cost coefficient in the cost model of heat exchanger between hot intermediate stream hm_p and cold process stream j_p
CA_{i_p,cm_p}	Variable cost coefficient in the cost model of heat exchanger between hot process stream i_p and cold intermediate stream cm_p
CA_{i_p,j_p}	Variable cost coefficient in the cost model of heat exchanger between hot stream i_p and cold stream j_p
CA_{i_p,n_p}	Variable cost coefficient in the cost model of cooler between hot stream i_p and cold utility n_p
CA_{m_p,j_p}	Variable cost coefficient in the cost model of heater between hot utility m_p and cold stream j_p
CF_{hm_p,j_p}	Fixed cost in the cost model of cooler between hot intermediate stream hm_p and cold process stream j_p
CF_{i_p,cm_p}	Fixed cost in the cost model of heat exchanger between hot process stream i_p and cold intermediate stream cm_p
CF_{i_p,j_p}	Fixed cost in the cost model of heat exchanger between hot process stream i_p and cold process stream j_p
CF_{i_p,n_p}	Fixed cost in the cost model of cooler between hot process stream i_p and cold utility n_p
CF_{m_p,j_p}	Fixed cost in the cost model of heater between hot utility m_p and cold process stream j_p
$CM^{i_p,j_p}_{\tilde{i}_p,\tilde{j}_p}$	Reassignment cost for existing heat exchanger which houses match (i_p,j_p) in the original HEN of plant p and houses different match $\left(\tilde{i}_p,\tilde{j}_p\right)$ in the multi-plant HEN
$CM^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p}$	Reassignment cost for existing cooler of type w_p which houses match (i_p,n_p) in the original HEN of plant p and houses different match $\left(\tilde{i}_p,\tilde{n}_p\right)$ in the multi-plant HEN
$CM^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p}$	Reassignment cost for existing heater of type l_p which houses match (m_p,j_p) in the original HEN of plant p and houses different match $\left(\tilde{m}_p,\tilde{j}_p\right)$ in the multi-plant HEN
CIM	Cost coefficient of intermediate fluid
CQ_{i_p,n_p}	Unit cost of cold utility n_p for cooling hot process stream i_p
CQ_{m_p,j_p}	Unit cost of hot utility m_p for heating cold process stream j_p
β	Exponent of heat transfer areas in variable cost terms in the cost models of heat exchanger, cooler and heater

5.1. *Cost models utilized for implementation of Strategy I*

In the present case, the cost models in Eq. (67) can be determined according to the following formulas listed in Eqs. (68)–(74).

$$
\begin{aligned}
TUC' = & \sum_{p=1}^{P} \sum_{i_p \in H^p} \sum_{n_p \in CU^p} CQ_{i_p,n_p} q_{i_p,n_p} \\
& + \sum_{p=1}^{P} \sum_{m_p \in HU^p} \sum_{j_p \in C^p} CQ_{m_p,j_p} q_{m_p,j_p}
\end{aligned}
\tag{68}
$$

$$
\begin{aligned}
ATCC = & \sum_{p=1}^{P} \sum_{(i_p,j_p) \in Y^p} \sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}} \sum_{k \in ST} \left[CF_{i_p,j_p} \sigma_{i_p,j_p,k}^{y_{i_p,j_p}} + CA_{i_p,j_p} \left(X_{i_p,j_p,k}^{y_{i_p,j_p}} \right)^{\beta} \right] \\
& + \sum_{p=1}^{P} \sum_{(i_p,n_p) \in CLM_p} \left[CF_{i_p,n_p} \sigma_{i_p,n_p} + CA_{i_p,n_p} \left(X_{i_p,n_p} \right)^{\beta} \right] \\
& + \sum_{p=1}^{P} \sum_{(m_p,j_p) \in HTM_p} \left[CF_{m_p,j_p} \sigma_{m_p,j_p} + CA_{m_p,j_p} \left(X_{m_p,j_p} \right)^{\beta} \right]
\end{aligned}
\tag{69}
$$

$$
\begin{aligned}
NTCC_1 = & \sum_{p=1}^{P} \sum_{i_p \in H^p} \sum_{cm_p \in CM^p} \sum_{k \in ST} \left[CF_{i_p,cm_p} \xi_{i_p,cm_p,k} + CA_{i_p,cm_p} \left(A_{i_p,cm_p,k} \right)^{\beta} \right] \\
& + \sum_{p=1}^{P} \sum_{hm_p \in HM^p} \sum_{j_p \in C^p} \sum_{k \in ST} \left[CF_{hm_p,j_p} \xi_{hm_p,j_p,k} + CA_{hm_p,j_p} \left(A_{hm_p,j_p,k} \right)^{\beta} \right]
\end{aligned}
\tag{70}
$$

$$
\begin{aligned}
NTCC_2 = & \sum_{p=1}^{P} \sum_{i_p \in H^p} \sum_{k \in ST} BY_{i_p} rz_{i_p,k} + \sum_{p=1}^{P} \sum_{j_p \in C^p} \sum_{k \in ST} BY_{j_p} rz_{j_p,k} \\
& + \sum_{p=1}^{P} \sum_{i_p \in H^p} BY_{i_p} rzhu_{i_p} + \sum_{p=1}^{P} \sum_{j_p \in C^p} BY_{j_p} rzcu_{j_p}
\end{aligned}
\tag{71}
$$

$$
NTCC_3 = 0 \tag{72}
$$

$$
NTCC_4 = 0 \tag{73}
$$

$$
TIMC = \sum_{p=1}^{P} \sum_{hm_p \in HM^p} CIM \cdot F_{hm_p} \tag{74}
$$

5.2. *Cost models utilized for implementation of Strategy II*

The cost models given in Subsection 5.1 are all applicable in the present case except *ATCC* and *NTCC*$_3$. In particular, Eqs. (69) and (72) should be replaced by the following formulas:

$$\begin{aligned} ATCC = & \sum_{p=1}^{P} \sum_{(i_p,j_p)\in Y^p} \sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}} \sum_{(\tilde{i}_p,\tilde{j}_p)\in Y^p} \sum_{k\in ST} \Bigg[CF_{\tilde{i}_p,\tilde{j}_p} \sigma^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} \\ & + CA_{\tilde{i}_p,\tilde{j}_p} \left(X^{y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} \right)^{\beta} \Bigg] \\ & + \sum_{p=1}^{P} \sum_{w_p=1}^{W_p} \sum_{(i_p,n_p)\in CL_p^{w_p}} \sum_{(\tilde{i}_p,\tilde{n}_p)\in CL_p^{w_p}} \Bigg[CF_{\tilde{i}_p,\tilde{n}_p} \sigma^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} \\ & + CA_{\tilde{i}_p,\tilde{n}_p} \left(X^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} \right)^{\beta} \Bigg] \\ & + \sum_{p=1}^{P} \sum_{l_p=1}^{L_p} \sum_{(m_p,j_p)\in HT_p^{l_p}} \sum_{(\tilde{m}_p,\tilde{j}_p)\in HT_p^{l_p}} \Bigg[CF_{\tilde{m}_p,\tilde{j}_p} \sigma^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} \\ & + CA_{\tilde{m}_p,\tilde{j}_p} \left(X^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} \right)^{\beta} \Bigg] \end{aligned} \tag{75}$$

$$\begin{aligned} NTCC_3 = & \sum_{p=1}^{P} \sum_{(i_p,j_p)\in Y^p} \sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}} \sum_{\substack{(\tilde{i}_p,\tilde{j}_p)\in Y^p \\ \tilde{i}_p\neq i_p \vee \tilde{j}_p\neq j_p}} \sum_{k\in ST} CM^{i_p,j_p}_{\tilde{i}_p,\tilde{j}_p} e^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} \\ & + \sum_{p=1}^{P} \sum_{w_p=1}^{W_p} \sum_{(i_p,n_p)\in CL_p^{w_p}} \sum_{\substack{(\tilde{i}_p,\tilde{n}_p)\in CL_p^{w_p} \\ \tilde{i}_p\neq i_p}} CM^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} e^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} \\ & + \sum_{p=1}^{P} \sum_{l_p=1}^{L_p} \sum_{(m_p,j_p)\in HT_p^{l_p}} \sum_{\substack{(\tilde{m}_p,\tilde{j}_p)\in HT_p^{l_p} \\ \tilde{j}_p\neq j_p}} CM^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} e^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} \end{aligned} \tag{76}$$

5.3. *Cost models utilized for implementation of Strategy III*

The cost models given in Subsection 5.1 are all applicable in the present scenario except *ATCC*, *NTCC*$_3$ and *NTCC*$_4$. The corresponding cost models can found below:

$$
\begin{aligned}
ATCC = & \sum_{p=1}^{P} \sum_{(i_p,j_p)\in Y^p} \sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}} \sum_{\left(\tilde{i}_p,\tilde{j}_p\right)\in Z^p} \sum_{k\in ST} \left[CF_{\tilde{i}_p,\tilde{j}_p} \sigma^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} \right. \\
& \left. + CA_{\tilde{i}_p,\tilde{j}_p} \left(X^{y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} \right)^{\beta} \right] \\
& + \sum_{p=1}^{P} \sum_{w_p=1}^{W_p} \sum_{(i_p,n_p)\in CL_p^{w_p}} \sum_{\left(\tilde{i}_p,\tilde{n}_p\right)\in ZCL_p^{w_p}} \left[CF_{\tilde{i}_p,\tilde{n}_p} \sigma^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} \right. \\
& \left. + CA_{\tilde{i}_p,\tilde{n}_p} \left(X^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} \right)^{\beta} \right] \\
& + \sum_{p=1}^{P} \sum_{l_p=1}^{L_p} \sum_{(m_p,j_p)\in HT_p^{l_p}} \sum_{\left(\tilde{m}_p,\tilde{j}_p\right)\in ZHT_p^{l_p}} \left[CF_{\tilde{m}_p,\tilde{j}_p} \sigma^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} \right. \\
& \left. + CA_{\tilde{m}_p,\tilde{j}_p} \left(X^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} \right)^{\beta} \right]
\end{aligned} \tag{77}
$$

$$
\begin{aligned}
NTCC_3 = & \sum_{p=1}^{P} \sum_{(i_p,j_p)\in Y^p} \sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}} \sum_{\substack{\left(\tilde{i}_p,\tilde{j}_p\right)\in Z^p \\ \tilde{i}_p \neq i_p \vee \tilde{j}_p \neq j_p}} \sum_{k\in ST} CM^{i_p,j_p}_{\tilde{i}_p,\tilde{j}_p} e^{i_p,j_p,y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} \\
& + \sum_{p=1}^{P} \sum_{w_p=1}^{W_p} \sum_{(i_p,n_p)\in CL_p^{w_p}} \sum_{\substack{\left(\tilde{i}_p,\tilde{n}_p\right)\in ZCL_p^{w_p} \\ \tilde{i}_p \neq i_p}} CM^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} e^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_p} \\
& + \sum_{p=1}^{P} \sum_{l_p=1}^{L_p} \sum_{(m_p,j_p)\in HT_p^{l_p}} \sum_{\substack{\left(\tilde{m}_p,\tilde{j}_p\right)\in ZHT_p^{l_p} \\ \tilde{j}_p \neq j_p}} CM^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p} e^{m_p,j_p,l_p}_{\tilde{m}_p,\tilde{j}_p}
\end{aligned} \tag{78}
$$

$$
\begin{aligned}
NTCC_4 = & \sum_{p=1}^{P} \sum_{(i_p,j_p)\in Z^p} \sum_{k\in ST} \left[CF_{i_p,j_p} + CA_{i_p,j_p} \left(A_{i_p,j_p,k} \right)^{\beta} \right] u_{i_p,j_p,k} \\
& + \sum_{p=1}^{P} \sum_{w_p=1}^{W_p} \sum_{(i_p,n_p)\in ZCL_p^{w_p}} \left[CF_{i_p,n_p} + CA_{i_p,n_p} \left(A_{i_p,n_p} \right)^{\beta} \right] u^{CU,w_p}_{i_p,n_p} \\
& + \sum_{p=1}^{P} \sum_{l_p=1}^{L_p} \sum_{(m_p,j_p)\in ZHT_p^{l_p}} \left[CF_{m_p,j_p} + CA_{m_p,j_p} \left(A_{m_p,j_p} \right)^{\beta} \right] u^{HU,l_p}_{m_p,j_p}
\end{aligned} \tag{79}
$$

6. Case studies: maximum-saving multi-plant HEN designs

As an illustrative example, let us consider three chemical plants (denoted as P1, P2 and P3), whose stream data and utility data are given in Tables 1 and 2 respectively (Lo et al., 2020). The parameters used in the proposed models are listed below:

- Lower bounds of heat duties (LQ_{i_p,j_p}, LQ_{hm_p,j_p}, LQ_{i_p,cm_p}, LQ_{i_p,n_p} and LQ_{m_p,j_p}) are all set to be: 30 kW.
- Lower bounds of flow fraction ($LRH_{i_p,j_p,k}$, $LRC_{i_p,j_p,k}$, $LRCM_{hm_p,j_p,k}$, $LRCMM_{i_p,cm_p,k}$, $LRHM_{i_p,cm_p,k}$ and $LRHMM_{hm_p,j_p,k}$) are all set to be: 0.1.
- Lower bounds of bypass flow fraction ($LRRC_{j_p,k}$, $LRRCU_{j_p}$, $LRRH_{i_p,k}$ and $LRRHU_{i_p}$) are all set to be: 0.1.
- Lower bounds of heat-transfer areas for the augmented units (LX_{i_p,j_p}, LX_{i_p,n_p} and LX_{m_p,j_p}) are all set to be: 1 m^2.
- Heat transfer coefficients of intermediate streams (h_{cm_p} and h_{hm_p}) are all set to be 1.5 kW/m$^{2\circ}$C.

Table 1 – Stream data of illustrative example.

Plant	Stream	TIN (°C)	TOUT (°C)	F_{cp} (kW/°C)	h $\left(\text{kW/m}^2{}^\circ\text{C}\right)$
P1	H1	150	40	7.0	1.2
	C1	60	140	9.0	1.6
	C2	110	190	8.0	1.0
P2	H1	200	70	5.5	1.5
	C1	30	110	3.5	1.1
	C2	140	190	7.5	1.2
P3	H1	370	150	3.0	1.4
	H2	200	40	5.5	1.1
	C1	110	360	4.5	1.3

Table 2 – Utility data of illustrative example.

Plant	Utility	TIN (°C)	TOUT (°C)	h $\left(\text{kW/m}^2\cdot{}^\circ\text{C}\right)$	CQ $\left(\$/\text{kW}\cdot\text{yr}\right)$
P1	Cooling water	25	30	1.2	200
	LP steam	200	200	1.5	375
	MP steam	250	250	1.8	575
	HP steam	300	300	2.3	775
	Hot oil	500	475	1.8	900
P2	Cooling water	25	30	1.2	250
	LP steam	200	200	1.5	400
	MP steam	250	250	1.8	600
	HP steam	300	300	2.3	800
	Hot oil	500	475	1.8	1000
P3	Cooling water	25	30	1.2	150
	LP steam	200	200	1.5	350
	MP steam	250	250	1.8	550
	HP steam	300	300	2.3	750
	Hot oil	500	475	1.8	850

- Based on an operation horizon of 10 years and a yearly interest rate of 5.85 %, an annualization factor (af) of 0.1349 is adopted in this study.
- The exponent of area in the variable cost term (β) is 0.83.
- The variable cost coefficients of the heat exchangers (CA_{i_p,j_p}, CA_{hm_p,j_p} and CA_{i_p,cm_p}) and those of coolers (CA_{i_p,n_p}) are all set to be: 380 $/m$^{1.66}$; The variable cost coefficients of heaters (CA_{m_p,j_p}) are all set to be: 700 $/m$^{1.66}$.
- The fixed costs of inner-plant units (CF_{i_p,j_p}, CF_{i_p,n_p} and CF_{m_p,j_p}) are all assumed to be: 10000 $. The fixed costs of inner-plant units (CF_{hm_p,j_p} and CF_{i_p,cm_p}) are all assumed to be 15000 $.
- The re-piping cost of every bypass (BY_{i_p} and BY_{j_p}) is 500 $.
- The reassignment cost for existing units ($CM_{\tilde{i}_p,\tilde{j}_p}^{i_p,j_p}$, $CM_{\tilde{i}_p,\tilde{n}_p}^{i_p,n_p,w_p}$ and $CM_{\tilde{m}_p,\tilde{j}_p}^{m_p,j_p,l_p}$) are all set to be: 2000 $.
- The cost coefficient of intermediate fluid (CIM) is set to be 10000 $ ·° C/kW.

The existing single-plant HENs are presented in Fig. 2. The minimum TACs of these HENs were found to be 316,565 $/year (P1), 56,294 $/year (P2) and 287,769 $/year (P3). Finally, it should be noted that the optimal solutions presented in the sequel were all obtained with solver BARON in GAMS 27.3 on a personal computer (Intel Core i7 6700; 16 G).

6.1. *Multi-plant retrofit designs obtained by applying Strategy I*

The proposed MINLP model can then be constructed according to the formulations described in Part A of the Supplementary material, and Subsections 4.1 and 5.1.

The optimal retrofit designs of multi-plant HENs for all possible coalitions can be generated by solving the proposed models. Fig. 3 is an example of various HEN structures generated with Strategy I, which shows the indirect interplant heat integration scheme obtained by revamping the single-plant HENs of P1, P2 and P3 (P1: heat sink; P2: heat source; P3: heat source). Notice also that the inner-plant matches between process streams, which are housed in the existing heat exchangers, are represented with vertically connected circles between streams without color background. The inner-plant matches that require augmented units are indicated with double circles, while the others are marked with single circles. The matches between process and intermediate streams, which are housed in the purchased new heat exchangers, are represented with vertically connected gray circles.

The total annual cost savings (TACSs) achieved by multi-plant retrofit designs for all possible coalitions are given in Table 3. It can be found that, when plant P1 is treated as heat sink and P2 and P3 heat sources, the greatest financial gain can be obtained for the three-plant coalition (see case 3). For the sake of completeness, the various costs and cost savings of multi-plant retrofit designs and the detailed assignments of new units are listed in Table C1 and Table C2 in Supplementary material, respectively.

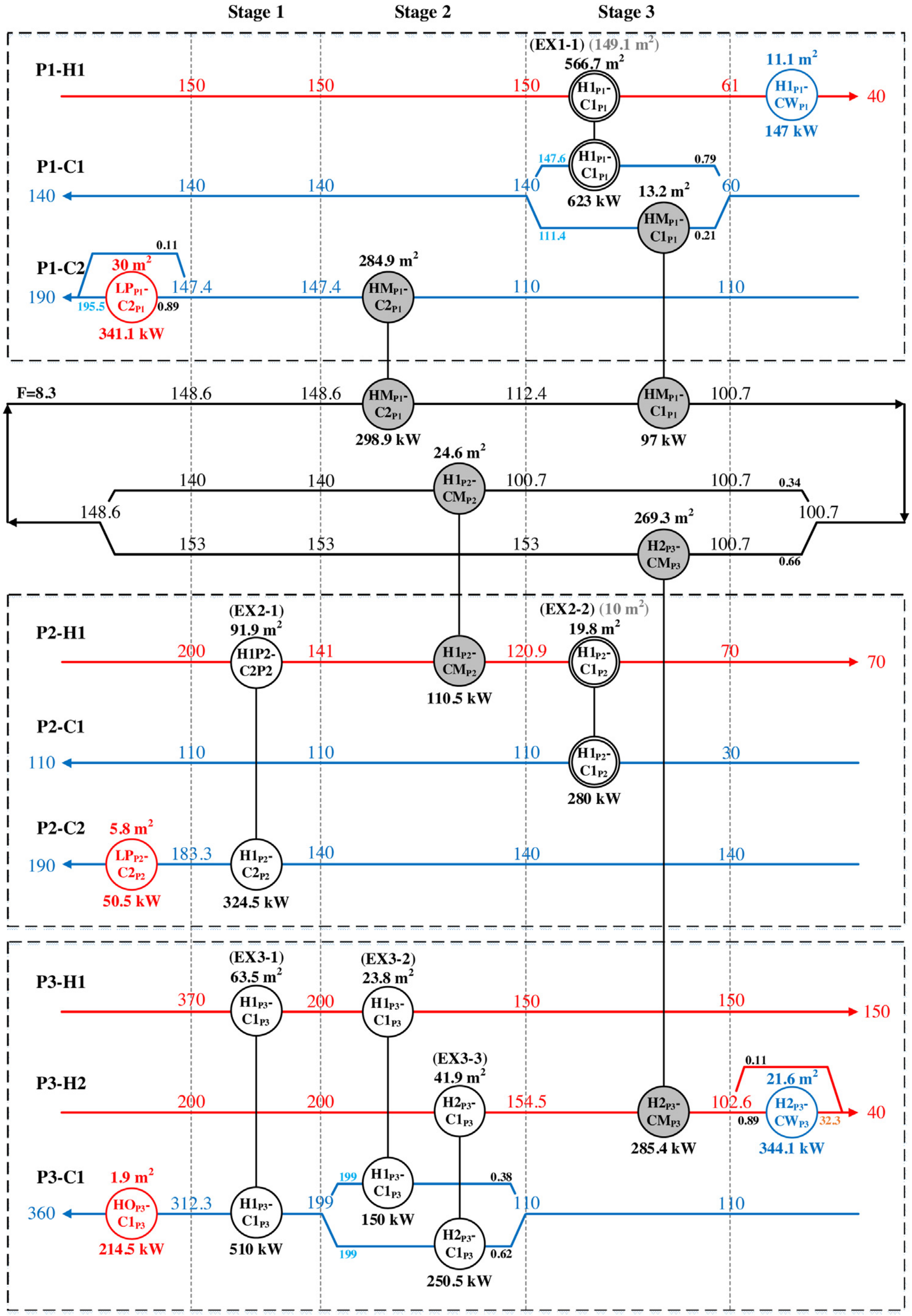

Fig. 3 – Indirect interplant heat integration scheme obtained by revamping the single-plant HENs of P1, P2 and P3 with Strategy I and Strategy II (P1: heat sink; P2: heat source; P3 : heat source).

Table 3 – TACs achieved for all indirect HEN integration cases with Strategy I.

Coalition	Case	P1	P2	P3	TACS ($/yr)
{P1, P2}	Case 1	Sink	Source	–	53, 876
	Case 2	Source	Sink	–	0
{P1, P3}	Case 1	Sink	–	Source	138, 019
	Case 2	Source	–	Sink	0
{P2, P3}	Case 1	–	Sink	Source	15, 669
	Case 2	–	Source	Sink	0
{P1, P2, P3}	Case 1	Sink	Sink	Source	138, 019
	Case 2	Sink	Source	Sink	53, 876
	Case 3	Sink	Source	Source	176, 702
	Case 4	Source	Sink	Sink	0
	Case 5	Source	Sink	Source	15, 669
	Case 6	Source	Source	Sink	0

Table 4 – TACs achieved for all integration schemes with Strategy II.

Coalition	Case	P1	P2	P3	TACS $\left(\$/\text{yr}\right)$
{P1, P2}	Case 1	Sink	Source	–	53, 876
	Case 2	Source	Sink	–	0
{P1, P3}	Case 1	Sink	–	Source	139, 834
	Case 2	Source	–	Sink	0
{P2, P3}	Case 1	–	Sink	Source	15, 669
	Case 2	–	Source	Sink	0
{P1, P2, P3}	Case 1	Sink	Sink	Source	139, 834
	Case 2	Sink	Source	Sink	53, 876
	Case 3	Sink	Source	Source	176, 702
	Case 4	Source	Sink	Sink	0
	Case 5	Source	Sink	Source	15, 669
	Case 6	Source	Source	Sink	0

Table 5 – TACs achieved for all integration schemes with Strategy III.

Coalition	Case	P1	P2	P3	TACS $\left(\$/\text{yr}\right)$
{P1, P2}	Case 1	Sink	Source	–	53876
	Case 2	Source	Sink	–	0
{P1, P3}	Case 1	Sink	–	Source	140, 159
	Case 2	Source	–	Sink	0
{P2, P3}	Case 1	–	Sink	Source	15, 669
	Case 2	–	Source	Sink	0
{P1, P2, P3}	Case 1	Sink	Sink	Source	140, 159
	Case 2	Sink	Source	Sink	53, 876
	Case 3	Sink	Source	Source	178, 702
	Case 4	Source	Sink	Sink	0
	Case 5	Source	Sink	Source	15, 669
	Case 6	Source	Source	Sink	0

6.2. *Multi-plant retrofit designs obtained by applying Strategy II*

Next, a MINLP model can be constructed according to the formulations described in Part A of the Supplementary material, and Subsections 4.2 and 5.2. The corresponding optimization results are shown in Fig. 3 and Table 4. Fig. 3 is also an example of various HEN structures generated in the present case, which shows the indirect interplant heat integration scheme obtained by revamping the single-plant HENs of P1, P2 and P3 with Strategy II (P1: heat sink; P2: heat source; P3: heat source). The symbols used in this figure follow exactly the same conventions described previously in Subsection 6.1. Notice from Table 4 that the revamped HEN in case 3 of three-plant coalition $\{P1, P2, P3\}$ (when plant P1 is viewed as heat sink and plants P2 and P3 heat sources) yields the greatest cost saving, whose value is the same as that of its counterpart in Table 3.

For the sake of completeness, additional results are provided in Part C of Supplementary material. The costs of multi-plant retrofit designs and the corresponding cost savings are detailed in Table C3. Table C4 shows the assignments of new units, while Table C5 shows the arrangements of existing units in multi-plant HEN for different coalitions. Since Strategy II allows reassignment of every existing heat exchanger, cooler and heater to any existing match of the same type within the same plant, greater TACSs may be achieved when compared with Strategy I. For example, the TACS of two-plant HEN for coalition $\{P1, P3\}$ obtained by Strategy II is greater than that obtained by Strategy I.

6.3. *Multi-plant retrofit designs obtained by applying Strategy III*

The MINLP model in this case can be constructed by combining the formulations proposed in Part A of the Supplementary material, and Subsections 4.3 and 5.3. The corresponding optimization results can be found in Fig. 4 and Table 5. Fig. 4 is an example of various HEN structures generated in the present case, which shows the indirect interplant heat integration scheme obtained by revamping the single-plant HENs of P1, P2 and P3 with Strategy III (P1: heat sink; P2: heat source; P3: heat source). The symbols used in this figure follow the same conventions described previously in Subsection 6.1. In addition, the new inner-plant matches between hot and cold process streams, which are housed in the new purchased units, are represented with vertically connected green circles. It can be found from Table 5 that, when plant P1 is regarded as heat sink and plants P2 and P3 heat source, the greatest financial gain can be obtained for three-plant coalition (see case 3).

For the sake of completeness, additional results are also provided in Part C of Supplementary material. The costs of multi-plant retrofit designs and the corresponding cost savings are detailed in Table C6. Table C7 shows the assignments of new units, while Table C8 shows the arrangements of existing units in multi-plant HEN for different coalitions. Since every existing unit is allowed to house any inner-plant match of the same type within the same plant and the inner-plant matches between hot and cold process streams can also be housed in new purchased units, greater TAC savings may be achieved when compared with the previous two strategies. For example, the maximum TACS of two-plant HEN achieved by coalition $\{P1, P3\}$ obtained by Strategy III is greater than that obtained by Strategy II (see Tables 4 and 5). Let us next consider another example, i.e., three-plant coalition $\{P1, P2, P3\}$, for the further discussion. Notice from Table C7 that the existing match $(H1_{P1}, C1_{P1})$ is housed in new purchased

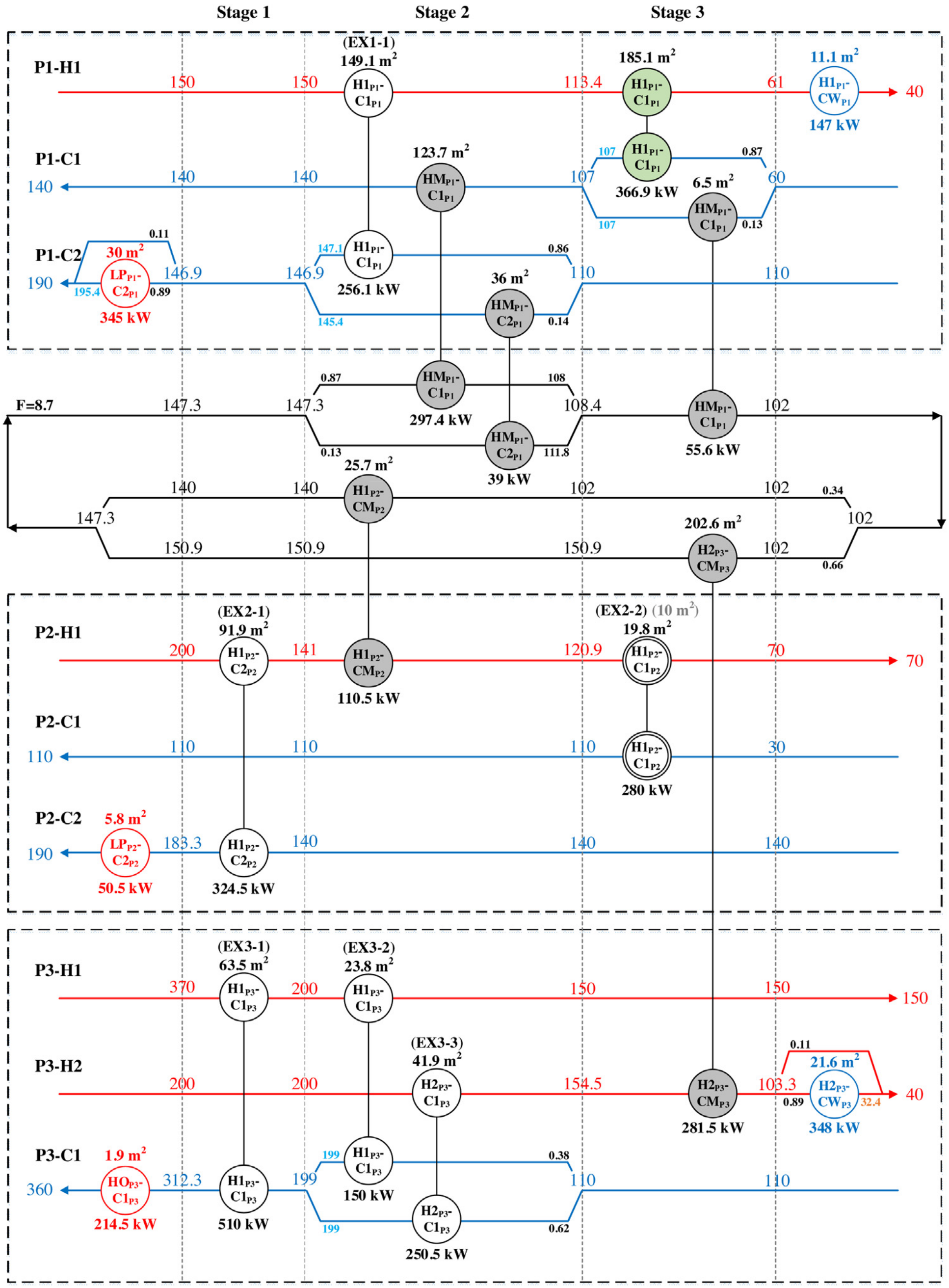

Fig. 4 – Indirect interplant heat integration scheme obtained by revamping the single-plant HENs of P1, P2 and P3 with Strategy III (P1: heat sink; P2: heat source; P3: heat source).

heat exchanger, while the existing heat exchanger in the original HEN of plant P1 houses the other match ($H1_{P1}$, $C2_{P1}$) in this multi-plant HEN retrofit design. This practice contributes to the cost saving of additional augmented units and the additional TACS when the present design is compared with its counterpart obtained with the previous strategy (see Table C3 and Table C6).

7. Case studies: benefit allocation plans

The above cost-optimal retrofit design of multi-plant HEN clearly lacks a benefit allocation plan. Only the maximum total saving of the entire system is determined by solving the aforementioned MINLP model, while the practical issues of distributing this TACS are not addressed at all. In the present work, this allocation problem is viewed as a cooperative game and all players of the game form a coalition. Typically, the *core* and *Shapley values* are used to characterize the reasonable and fair solution(s) for distributing the financial benefit within the coalition (Branzei et al., 2008). The core depicts the feasible region for an allocation scheme to be stable based on the principles of individual rationality, group rationality, coalition rationality and no cross subsidization. The Shapley values, on the other hand, determine the proportions of TACS that the participating plants (or players) can obtain. For the sake of completeness, a brief summary of the above two concepts are provided in part D of the Supplementary material. Since further discussions of the above materials can also be found in a companion paper published just recently (Lo et al., 2020), the Shapley values for the three-plant example described in the previous section are presented directly in the sequel without detailed computation steps simply for the sake of brevity.

7.1. *Allocation plans for retrofit designs obtained with Strategy I*

Firstly, the Shapley values for the present example can be generated by substituting the TACSs obtained for all possible coalitions. For the three-plant coalition {P1, P2, P3}, the allocated benefits were found to be 85,660 $/yr, 24,486 $/yr and 66,557 $/yr, respectively. The corresponding core and Shapley value have been plotted in a 2-D plot together. Since the Shapley-value mark is located inside the core, it can be deduced that the coalition is stable.

7.2. *Allocation plans for retrofit designs obtained with Strategy II*

The Shapley values for the present case, which can be determined on the basis of TACSs of all possible coalitions, can also be calculated according to the computation procedure given in Part D in the Supplementary material. The allocated benefits for the three-plant coalition {P1, P2, P3} can be determined to be 85,963 $/yr, 23,880 $/yr and 66,859 $/yr, respectively. A plot of the core and Shapley value also reveals that the allocation scheme is rational enough so as to maintain the stability of cooperative coalition.

7.3. *Allocation plans for retrofit designs obtained with Strategy III*

The Shapley values for the three-plant coalition {P1, P2, P3} in the present scenario were found to be 86,683 $/yr, 24,438 $/yr and 67,580 $/yr, respectively. This coalition is also believed to be stable based on the corresponding plots of Shapley values and core.

8. Conclusions

A rigorous procedure for generating multi-plant HEN retrofit designs via intermediate fluid(s) and the corresponding benefit allocation plans is presented in this paper. Several conclusions can be drawn and they are listed below:

- The proposed MINLP models can be systematically constructed for synthesizing the TSHI retrofit designs which are both energy and cost efficient.
- Indirect interplant heat integration approaches, which are applied in the multi-plant HEN retrofit designs, are demonstrated to be feasible in the illustrative three-plant example.
- The fair and reasonable benefit allocation schemes can be obtained according to conventional Shapley values and verified by core test.
- It can also be deduced from the results of the presented example that the proposed design strategy is clearly practicable in any other realistic environment.

Acknowledgement

This work is supported by the ministry of science and technology of the Taiwan government under grant 109-2221-E-006-085.

Appendix A. Supplementary data

Supplementary material related to this article can be found, in the online version, at doi:https://doi.org/10.1016/j.cherd.2021.07.032.

References

Bagajewicz, M., Rodera, H., 2002. Multiple plant heat integration in a total site. AIChE J. 48 (10), 2255–2270.
Boldyryev, S., Shamraev, A.A., Shamraeva, E.O., 2021. The design of the total site exchanger network with intermediate heat carriers: theoretical insights and practical application. Energy 223, 120023.
Bradley, G.H., Brown, G.G., Graves, G.W., 1977. Design and implementation of large scale primal transshipment algorithms. Manage. Sci. 24 (1), 1–34.
Branzei, R., Dimitrov, D., Tijs, S., 2008. Models in Cooperative Game Theory, 2nd ed. Springer, Berlin.
Chang, C., Wang, Y., Feng, X., 2015. Indirect heat integration across plants using hot water circles. Chin. J. Chem. Eng. 23 (6), 992–997.
Chang, C., Chen, X., Wang, Y., Feng, X., 2017. Simultaneous optimization of multi-plant heat integration using intermediate fluid circles. Energy 121, 306–317.
Chang, H.H., Chang, C.T., Li, B.H., 2018. Game-theory based optimization strategies for stepwise development of indirect interplant heat integration plans. Energy 148, 90–111.
Cheng, S.L., Chang, C.T., Jiang, D., 2014. A game-theory based optimization strategy to configure inter-plant heat integration schemes. Chem. Eng. Sci. 118, 60–73.
Ciric, A.R., Floudas, C.A., 1989. A retrofit approach for heat exchanger networks. Comput. Chem. Eng. 13 (6), 703–715.
Ciric, A.R., Floudas, C.A., 1990. A comprehensive optimization model of the heat exchanger network retrofit problem. Heat Recovery Syst. CHP 10 (4), 407–422.
Fernandez, F.R., Hinojosa, M.A., Puerto, J., 2002. Core solutions in vector-valued games. J. Optim. Theory Appl. 112, 331–360.
Floudas, C.A., Ciric, A.R., Grossmann, I.E., 1986. Automatic synthesis of optimum heat exchanger network configurations. AIChE J. 32 (2), 276–290.
Grabisch, M., Xie, L., 2007. A new approach to the core and Weber set of multichoice games. Math. Method Oper. Res. 66 (3), 491–512.

Gundersen, T., Grossmann, I.E., 1990. Improved optimization strategies for automated heat exchanger network synthesis through physical insights. Comput. Chem. Eng. 14 (9), 925–944.

Hiete, M., Ludwig, J., Schultmann, F., 2012. Intercompany energy integration. J. Ind. Ecol. 16 (5), 689–698.

Hong, X., Liao, Z., Sun, J., Jiang, B., Wang, J., Yang, Y., 2019. Transshipment type heat exchanger network model for intra- and inter-plant heat integration using process streams. Energy 178, 853–866.

Jin, Y., Chang, C.T., Li, S., Jiang, D., 2018. On the use of risk-based Shapley values for cost sharing in interplant heat integration programs. Appl. Energy 211, 904–920.

Klemes, J.J., 2013. Handbook of Process Integration (PI): Minimisation of Energy and Water Use, Waste and Emissions. Woodhead/Elsevier, Cambridge, UK.

Kralj, A.K., 2008. Heat integration between two biodiesel processes using a simple method. Energy Fuels 22 (3), 1972–1979.

Laukkanen, T., Tveit, T.M., Fogelholm, C.J., 2012. Simultaneous heat exchanger network synthesis for direct and indirect heat transfer inside and between processes. Chem. Eng. Res. Des. 90 (9), 1129–1140.

Liew, P.Y., Alwi, S.R.W., Klemes, J.J., Varbanov, P.S., Manan, Z.A., 2014. Algorithmic targeting for total site heat integration with variable energy supply/demand. Appl. Therm. Eng. 70 (2), 1073–1083.

Liew, P.Y., Theo, W.L., Alwi, S.R.W., Lim, J.S., Manan, Z.A., Klemes, J.J., Varbanov, P.S., 2017. Total site heat integration planning and design for industrial, urban and renewable systems. Renew. Sustain. Energ. Rev. 68, 964–985.

Linnhoff, B., Hindmarsh, E., 1983. The pinch design method for heat exchanger networks. Chem. Eng. Sci. 38 (5), 745–763.

Liu, L., Song, H., Zhang, L., Du, J., 2018. Heat-integrated water allocation network synthesis for industrial parks with sequential and simultaneous design. Comput. Chem. Eng. 108, 408–424.

Liu, L., Sheng, Y., Zhuang, Y., Zhang, L., Du, J., 2020. Multiobjective optimization of interplant heat exchanger networks considering utility steam supply and various locations of interplant steam generation/utilization. Ind. Eng. Chem. Res. 59 (32), 14433–14446.

Lo, F.Y., Zhu, Y.X., Chang, C.T., 2020. Optimal retrofit of a multi-plant heat exchanger network with a fair benefit allocation plan. Ind. Eng. Chem. Res. 59, 18088–18105.

Papoulias, S.A., Grossmann, I.E., 1983. A structural optimization approach in process synthesis—II: heat recovery networks. Comput. Chem. Eng. 7 (6), 707–721.

Ponce-Ortega, J.M., Jiménez-Gutiérrez, A., Grossmann, I.E., 2008. Simultaneous retrofit and heat integration of chemical processes. Ind. Eng. Chem. Res. 47 (15), 5512–5528.

Shapley, L.S., 1953. A value for n-person games. Contrib. Theory Games 2 (28), 307–317.

Smith, R., Jobson, M., Chen, L., 2010. Recent development in the retrofit of heat exchanger networks. Appl. Therm. Eng. 30 (16), 2281–2289.

Song, R., Chang, C., Tang, Q., Wang, Y., Feng, X., El-Halwagi, M.M., 2017. The implementation of inter-plant heat integration among multiple plants. Part II: the mathematical model. Energy 135, 382–393.

Sorsak, A., Kravanja, Z., 2004. MINLP retrofit of heat exchanger networks comprising different exchanger types. Comput. Chem. Eng. 28 (1–2), 235–251.

Tan, R.R., Andiappan, V., Wan, Y.K., Ng, R.T.L., Ng, D.K.S., 2016. An optimization-based cooperative game approach for systematic allocation of costs and benefits in interplant process integration. Chem. Eng. Res. Des. 106, 43–58.

Tao, R., Liu, L., Gu, S., Zhuang, Y., Zhang, L., Du, J., 2020. Flexible synthesis of inter-plant heat exchanger networks considering the operation of intermediate circles. Chem. Eng. Trans. 81, 13–18.

Tian, Y., Jin, Y., Li, S., 2020a. Modeling and optimization of inter-plant indirect heat exchanger networks by a difference evolutionary algorithm. Chem. Eng. Sci. 227, 115924.

Tian, Y., Wang, S., Liu, K., Li, S., 2020b. Two-plant direct heat integration with safety redundancy based on a bilevel algorithm. Chem. Eng. Sci. 222, 115662.

Wang, Y., Feng, X., 2016. Heat integration across plants considering distance factor. Adv. Energy Syst. Eng., 621–648.

Wang, Y., Chang, C., Feng, X., 2015. A systematic framework for multi-plants heat integration combining direct and indirect heat integration methods. Energy 90, 56–67.

Wang, Y., Wan, Z., Chang, C., Feng, X., 2020. A game theory based method for inter-plant heat integration considering cost allocation. Chin. J. Chem. Eng. 28 (6), 1652–1660.

Yee, T.F., Grossmann, I.E., 1990. Simultaneous optimization models for heat integration—II. Heat exchanger network synthesis. Comput. Chem. Eng. 14 (10), 1165–1184.

Yee, T.F., Grossmann, I.E., 1991. A screening and optimization approach for the retrofit of heat-exchanger networks. Ind. Eng. Chem. Res. 30 (1), 146–162.