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Model based approach to synthesize spare-supported cleaning schedules for existing heat exchanger networks

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ABSTRACT

Almost every modern chemical process is equipped with a heat-exchanger network (HEN) for optimal energy recovery. However, as time goes on after startup, fouling on the heat-transfer surface in an industrial environment is unavoidable. If the heat exchangers in an operating plant are not cleaned regularly, the targeted thermal efficiency of HEN can only be sustained for a short period of time. To address this practical issue, several mathematical programming models have already been developed to synthesize online cleaning schedules. Although the total utility cost of a HEN could be effectively reduced accordingly, any defouling operation still results in unnecessary energy loss due to the obvious need to temporarily take the unit to be cleaned out of service. The objective of the present study is thus to modify the available model so as to appropriately assign spares to replace them. Specifically, two binary variables are adopted to respectively represent distinct decisions concerning each online exchanger in a particular time interval, i.e., whether it should be cleaned and, if so, whether it should be substituted with a spare. The optimal solution thus includes not only the cleaning schedule but also the total number of spares, their capacities and the substitution schedule. Finally, the optimization results of a series of case studies are also presented to verify the feasibility of the proposed approach.

1. Introduction

In a chemical manufacturing process, efficient energy recovery and reuse is usually the key to minimizing the total operating cost, while the heat exchanger network (HEN) is a viable vehicle for achieving such a purpose. After putting the units in a HEN in service, the solid impurities in process streams may be deposited continuously on the heat-transfer surfaces and, thus, the overall performance of HEN tends to deteriorate over time. This fouling problem can be abated by cleaning all heat exchangers as a part of the overall maintenance (or checkup) program during plant shutdown. However, if it is also possible to clean at least a portion of the online units when the normal production is still in progress, then a proper schedule must be stipulated to maximize the implied cost saving.

A programming approach has often been adopted in the past to produce the aforementioned HEN cleaning schedules for energy conservation. To this end, Smaïli et al. (1999) first constructed a mixed integer nonlinear programming (MINLP) model for the thin-juice preheat train in a sugar refinery. Since the global solution of such a model cannot always be obtained, several additional studies have been carried out to address the related computation issues. Georgiadis et al. (1999) tried to developed a mixed integer linear program (MILP) via linearization of the nonlinear constraints so as to produce the near-optimum schedules efficiently, while Georgiadis and Papageorgiou (2000) later studied solution strategies of the corresponding MINLP models. Alle et al. (2002) then solved a few example problems successfully with the outer approximation algorithm. Smaïli et al. (2002) subsequently applied the simulated annealing, threshold accepting and backtracking threshold accepting algorithms to solve the models they first developed. Again for the same objective of achieving an approximate global optimum efficiently, Lavaja and Bagajewicz (2004) formulated a new MILP model via linearization to synthesize the cleaning schedules. Their solutions were compared with those obtained in Smaïli et al. (2002) and it was found that both yielded similar schedules and roughly the same total annual costs. On the other hand, Markowski and Urbaniec (2005) suggested using a graphic method to analyze the effects of fouling on the exit temperatures of every unit in a HEN and to manipulate the cleaning schedules accordingly.

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Nomenclature

Sets

E The set of all exchanger labels in the given HEN
I The set of all hot-stream labels in the given HEN
J The set of all cold-stream labels in the given HEN
P_k The set of all period labels in year k of the time horizon

Variables

A_{sp} Heat-transfer area of a spare exchanger (m^2)
$a_{i,j,k,p}^{fm,tp}$ The overall heat-transfer coefficient determined according to fouling model fm $\in \{L, E\}$ at time point tp $\in \{bcp, ecp, bop, eop\}$ during period p ($p \geq 2$) in scenario (i) if exchanger $(i, j) \in$ E is last cleaned during period k and $1 \leq k < p$ (kW/m^2 K)
$c_{i,j,p}^{fm,tp}$ The overall heat-transfer coefficient determined according to fouling model fm $\in \{L, E\}$ at time point tp $\in \{bcp, ecp, bop, eop\}$ during period p in scenario (iii) (kW/m^2 K)
$Eu_{j,p}^{H}, Eu_{i,p}^{C}$ Estimates of the total hot and cold utility consumption levels needed respectively by cold stream j $\in$ J and hot stream i $\in$ I in period p (kW-mon)
N_{sp} Total number of spares
$Qu_{j,p}^{H,tp}, Qu_{i,p}^{C,tp}$ The hot and cold utility consumption rates needed respectively by cold stream j $\in$ J and hot stream i $\in$ I at time point tp $\in \{bcp, ecp, bop, eop\}$ in period p (kW)
$r_{i,j}$ The fouling resistance of heat exchanger $(i, j) \in$ E (m^2 K/kW)
$T_{in,i,p}^{H,tp}, T_{out,i,p}^{H,tp}$ The inlet and outlet temperatures of hot stream i $\in$ I of exchanger $(i, j) \in$ E at time point tp $\in \{bcp, ecp, bop, eop\}$ in period p (K)
$T_{in,j,p}^{C,tp}, T_{out,j,p}^{C,tp}$ The inlet and outlet temperatures of cold stream j $\in$ J of exchanger$(i, j) \in$ E at time point tp $\in \{bcp, ecp, bop, eop\}$ in period p (K)
$TL_{i,p}^{H,tp}, TL_{j,p}^{C,tp}$ The outlet temperatures of hot and cold streams respectively from the last heat exchangers on streams i $\in$ I and j $\in$ J at time point tp $\in \{bcp, ecp, bop, eop\}$ in period p (K)
$U_{i,j,p}^{fm,tp}$ The overall heat-transfer coefficients of exchanger(i, j) at time point tp $\in \{bcp, ecp, bop, eop\}$ in period p determined according to fouling model fm $\in \{L, E\}$ (kW/m^2 K)
$X_{i,j,p}$ A binary variable used to denote whether or not a spare is adopted to replace exchanger $(i, j) \in E$ during period p
$Y_{i,j,p}$ A binary variable used to denote whether or not exchangerer $(i, j) \in E$ is cleaned during period p

Parameters

$A_{i,j}$ The heat-transfer area of exchanger $(i, j) \in$ E (m^2)
$bsp_{i,j,p}^{fm,tp}$ The overall heat-transfer coefficient determined according to fouling model fm $\in \{L, E\}$ at time point tp $\in \{bcp, ecp, bop, eop\}$ during period p in scenario (ii) if a spare is adopted to replace exchanger $(i, j) \in$ E (kW/m^2 K)
C_i^H, C_j^C The heat capacities of hot stream i $\in$ I and cold stream j $\in$ J (kJ/kg-K)
C_p^{HU}, C_p^{CU} The unit costs of heating and cooling utilities in period p ($/kJ)
C_{cl}, C_{cl}^{sp} The cleaning costs of a heat exchanger and a spare ($/cleaning)
C_{sp} Annualized cost coefficient for the capital cost of heat exchanger $\left(\$/m^{1.6}\text{yr}\right)$
f_c The duration of a defouling sub-period (mon)
F_i^H, F_j^C The mass flow rates of hot stream i $\in$ I and cold stream j $\in$ J (kg/s)
$K_{i,j}$ The characteristic fouling speed of exchanger $(i, j) \in$ E (mon^{-1})
$R_{i,j}$ The ratio between the products of mass flow rate and heat capacity of the cold and hot streams in heat exchanger $(i, j) \in$ E
$\dot{r}_{i,j}$ The constant fouling rate of exchanger $(i, j) \in$ E (m^2 K/mon kW)
$r_{i,j}^{\infty}$ The asymptotic maximum fouling resistance of exchanger $(i, j) \in$ E (m^2 K/kW)
t_f The overall time horizon (mon)
TT_i^H, TT_j^C The target temperatures of hot stream i $\in$ I and cold stream j $\in$ J (K)
$U_{i,j}^{cl}, U_{sp}^{cl}$ The overall heat-transfer coefficients of exchanger $(i, j) \in$ E and spare exchanger when the heat-transfer surface is clean (kW/m^2 K)
τ_p The length of period p (mon)
η_{cl} Efficiency of cleaning operation
η^H, η^C The heat-transfer efficiencies in heater and cooler respectively

Superscripts
bcp The time point at the beginning of cleaning sub-period
bop The time point at the beginning of operation sub-period
ecp The time point at the end of cleaning sub-period
eop The time point at the end of operation sub-period
E The exponential fouling model
L The linear fouling model

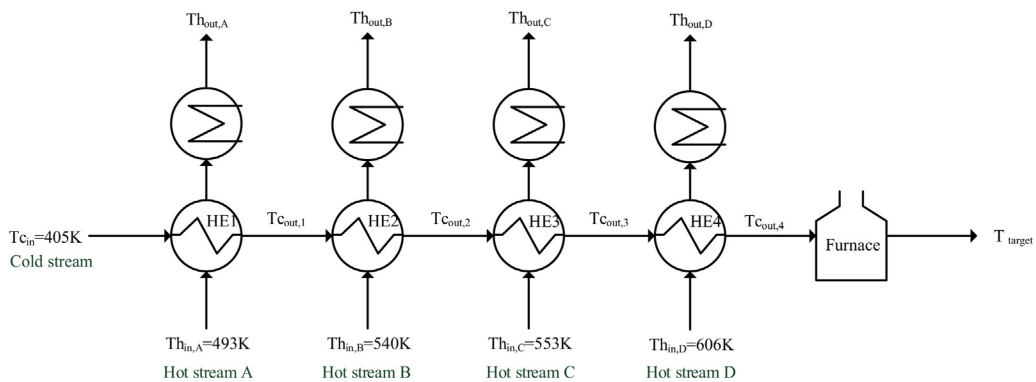

Fig. 1. Flow diagram of HEN considered in Example 1.

Table 1
Stream data of Example 1.

Stream	Inlet Temp. (K)	Target Temp. (K)	Heat Capacity Flow Rate (kW/K)
C	405	606	175.2
HA	493	353	49.98
HB	540	353	27.28
HC	553	353	138.48
HD	606	353	140.3

Assis et al. (2013) proposed to apply heuristic rules to roughly predict the performance of each heat exchanger before solving the mathematical programs so as to avoid trapping in the local optimum, while Gonçalves et al. (2014) also adopted the so-called recursive heuristics to facilitate effective convergence to the optimal cleaning schedule.

Other than the above studies on solution strategies, a few practical issues were also addressed in realistic applications. Sanaye and Niroomand (2007) produced the optimal HEN cleaning schedule for the urea and ammonia units by minimizing the operating cost using a numerical optimization method. Ishiyama et al. (2010) synthesized the cleaning schedule for the crude preheating train with special emphasis on maintaining a stable feed temperature of the desalting unit, while Ishiyama et al. (2011) considered different cleaning models for fouling and aging on the heat-transfer surface. Finally, in a grassroot design, Xiao et al. (2010) developed a programming approach to generate both a HEN structure and the corresponding cleaning schedule by minimizing the total annual cost.

One can observe from the above literature that, although the total utility cost of a HEN could be effectively reduced with cleaning, every defouling operation still results in unnecessary energy loss due to the obvious need to temporarily take the designated unit out of service. This undesired side effect may be circumvented by introducing a spare heat exchanger to replace the original one when cleaning operation is in progress. Therefore, the objective of this study is to modify the existing MINLP model so as to synthesize the optimal spare-supported cleaning schedules. This improved model is presented in detail in the sequel and two examples are then provided to demonstrate its feasibility.

2. Problem description

As mentioned before, the heat-transfer efficiency of one or more unit in a HEN could seriously deteriorate due to fouling. Since the target temperatures of all hot and cold streams cannot be reached in this situation, extra utilities must be consumed so as to meet the design conditions. To fix ideas, let us consider the simple flow diagram presented in Fig. 1, and the corresponding stream data and design specifications given in Tables 1 and 2 respectively. The furnace in this process is obviously adopted to raise the cold outlet temperature of HE4 (494 K) to the final temperature of 606 K, while the cooler on each hot stream may be either actually present or viewed as a part of the cooling capability embedded in a downstream unit. If there is a need to remove an exchanger temporarily for cleaning purpose, then its hot and cold streams must be diverted respectively via separate bypasses to the next units in HEN. It is assumed that extra capacities have been built into the aforementioned furnace and coolers so that they can be manipulated to handle the additional duties when one or more exchanger is taken offline.

Table 2
Design specifications of heat exchangers in Example 1.

Heat Exchanger	$U_{i,j}^{cl}$(kW/m^2 K)	$A_{i,j}$(m^2)	Cold Stream Inlet and Outlet Temp. (K)	Hot Stream Inlet and Outlet Temp. (K)	Heat Duty (kW)
HE1	0.5	43.2	405/413	493/463	1401.6
HE2	0.5	26.7	413/421	540/492	1401.6
HE3	0.5	110.7	421/452	553/514	5431.2
HE4	0.5	138.2	452/494	606/553	7358.4

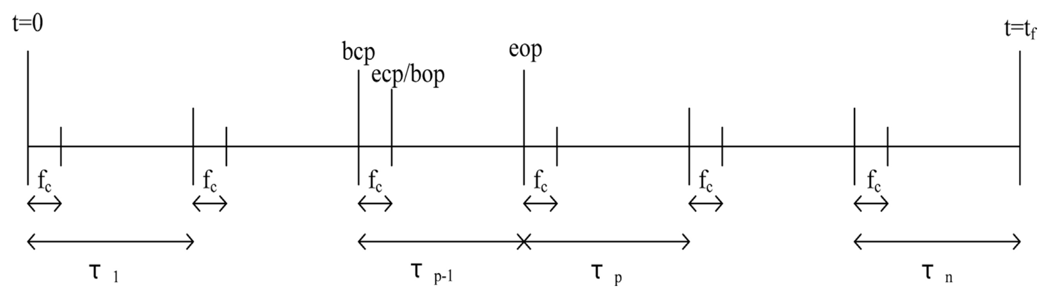

Fig. 2. Time horizon partitioning.

Since the fouling-related costs are affected by a large number of contributing factors, e.g., the given HEN structure, the duration of each cleaning operation, the corresponding heat load that must be taken out of service, the capacity of spare used to replace this service and the required capital investment, etc., a programming approach is needed to synthesize the optimal defouling and spare substitution schedules simultaneously so as to minimize the total annual cost (TAC). For model simplicity, the following assumptions have been adopted in the present work:

1. All cleaning durations are fixed at a predetermined constant value;
2. All cleaning operations cost the same and this value is given a priori;
3. Only identical spares are allowed.

The inputs to the proposed mathematical programming model should include: the original HEN design data (such as those given in Table 1, Table 2 and Fig. 1), the heat-transfer models (i.e., the fouling resistance function, the cleaning efficiency and the heat-recovery efficiencies of utility heaters and coolers), the time-horizon partition scheme (i.e., the total length of time horizon, the time interval between two consecutive cleaning operations and a constant time duration allocated for every defouling operation), and also various cost models (i.e., the unit costs of hot and cold utilities, the operating cost for cleaning an exchanger, and the capital cost model for a spare). Solving the proposed model should produce the following results: (1) the optimal cleaning and spare substitution schedules, (2) the number of spares to be purchased and their heat-transfer areas, and (3) the total utility cost, the total cleaning cost and the total capital investment.

3. Time horizon partitioning

In this work, the maximum length of time horizon that can be considered for schedule synthesis (say t_f) is the duration in months between the ending and beginning instances of two consecutive planned plant shutdowns. To simplify calculation, the entire duration of a cleaning schedule for a given HEN is set to be coincided with this time interval. In addition, the schedule horizon $[0, t_f]$ is partitioned into n different periods according to Fig. 2 and each is further divided into two intervals for performing the cleaning and heat-exchange operations respectively. For simplification purpose, the aforementioned time periods are fixed, i.e.

$$\tau_1 = \tau_2 = \cdots = \tau_n = \tau \tag{1}$$

$$t_f = \sum_{p=1}^{n} \tau_p = n\tau \tag{2}$$

where, τ_p denotes the length of period p ($p = 1, 2, \cdots, n$) and τ is a given constant. Also, it is assumed that the durations of all sub-periods required for defouling (f_c) are the same and their values can be determined in advance. Thus, within each partitioned period, four time points should be identified to facilitate accurate presentation of the proposed model, i.e., *bcp* (beginning of cleaning sub-period), *ecp* (end of cleaning sub-period), *bop* (beginning of operation sub-period), and *eop* (end of operation sub-period). Finally, for the sake of computation convenience, let us further assume that

$$t_f = 12 \times I_{tf} \tag{3}$$

where I_{tf} is a nonnegative integer. To facilitate clear explanation of the multi-year cleaning schedule, let us define a period set for each year, i.e.

$$\mathrm{P}_k = \left\{ p \mid p \text{ is the numerical label of a period in year } k \text{ of the cleaning schedule} \right\} \tag{4}$$

4. Binary variables to facilitate exchanger-cleaning selections

For illustration convenience, let us introduce the following two label sets to collect and classify the process streams in a given HEN:

$$\mathrm{I} = \left\{ i \mid i \text{ is the label of a hot stream in a given HEN} \right\} \tag{5}$$

$$\mathrm{J} = \left\{ j \mid j \text{ is the label of a cold stream in a given HEN} \right\} \tag{6}$$

In addition, the heat exchangers in this HEN can be written as

$$\mathrm{E} = \left\{ (i,j) \mid (i,j) \text{ denotes an exchanger in a given HEN, } i \in \mathrm{I}, j \in \mathrm{J} \right\} \tag{7}$$

Therefore, the selections of exchangers to be cleaned can be expressed accordingly with the following binary variable:

$$Y_{i,j,p} = \begin{cases} 1 \text{ if heat exchanger } (i,j) \text{ is cleaned in period } p \\ 0 \text{ otherwise} \end{cases} \tag{8}$$

where, $(i,j) \in \mathrm{E}$ and $p = 1, 2, \cdots, n$. Finally, for formulation simplicity, let us set $Y_{i,j,0} = 0$ in the proposed model.

5. Binary variables to represent spare-substitution options

The need to consider a spare only arises after making the decision to remove and clean an online unit from HEN. All such options can be represented with another set of binary variables and the corresponding logic constraints, i.e.

$$X_{i,j,p} = \begin{cases} 1 \text{ if heat exchanger } (i,j) \text{ is replaced with a spare in period } p \\ 0 \text{ otherwise} \end{cases} \tag{9}$$

$$\left(1 - Y_{i,j,p}\right) + X_{i,j,p} \le 1 \tag{10}$$

where, $(i,j) \in \mathrm{E}$ and $p = 1, 2, \cdots, n$. To set the upper limit for capital investment, it may also be necessary to impose the following inequality constraints:

$$\sum_{(i,j) \in \mathrm{E}} X_{i,j,p} \le N_{sp} \le \sum_{(i,j) \in \mathrm{E}} Y_{i,j,p} \tag{11}$$

where, N_{sp} is the maximum number of purchased spares and it is a given model parameter.

6. Fouling models

As a result of fouling during the normal operation, the overall heat-transfer coefficient of every exchanger in HEN may decrease with time according to the following formula (Lavaja and Bagajewicz, 2004)

$$U_{i,j}(t) = \left[\frac{1}{U_{i,j}^{cl}} + r_{i,j}(t) \right]^{-1} \tag{12}$$

where, $(i,j) \in \mathrm{E}$; $U_{i,j}(t)$ is the overall heat-transfer coefficient of exchanger (i,j) at time t and $U_{i,j}^{cl}$ denotes the corresponding value when the heat-transfer surface is clean. The time function $r_{i,j}(t)$ is the fouling resistance of exchanger (i,j) at time t, which can be expressed with either a linear or exponential model:

$$r_{i,j}(t) = \dot{r}_{i,j} t \tag{13}$$

$$r_{i,j}(t) = r_{i,j}^{\infty}[1 - \exp(-K_{i,j} t)] \tag{14}$$

where, $\dot{r}_{i,j}$ denotes the constant fouling rate, $r_{i,j}^{\infty}$ is the asymptotic maximum fouling resistance, $K_{i,j}$ is the characteristic fouling speed, and all of them are given model parameters.

7. Model constraints

Only the model constraints established on the basis of the *linear fouling* assumption are presented in the present section for the sake of brevity, while the corresponding formulations derived from the exponential model can be found in Appendix A. Basically both sets of constraints can be established by introducing the spare-substitution options into the existing model framework developed by Smaïli et al. (1999, 2002) and Lavaja and Bagajewicz (2004). Note that the superscripts L and tp in the subsequent discussions are used to denote a variable (or parameter) associated with the linear model and time point $tp \in \left\{ bcp, ecp, bop, eop \right\}$ respectively. An accurate description of the corresponding model can be presented accordingly in the sequel:

7.1. Overall heat transfer coefficients

Exactly three scenarios should be considered in modeling the overall heat-transfer coefficient in period p:

(i) Exchanger (i, j) is not cleaned during period p ($p \geq 2$), but in at least one of the prior periods defouling operation has been performed, i.e., $Y_{i,j,p} = 0$ and $\prod_{k=1}^{p-1}(1 - Y_{i,j,k}) = 0$ for $p = 2, 3, \cdots, n$.
(ii) Exchanger (i, j) is cleaned in period p, i.e., $Y_{i,j,p} = 1$ for $p = 1, 2, \cdots, n$.
(iii) Exchanger (i, j) has never been cleaned since period 1, i.e., $Y_{i,j,1} = Y_{i,j,2} = \cdots = Y_{i,j,p} = 0$ for $p = 1, 2, \cdots, n$.

The overall heat-transfer coefficient of exchanger (i, j) at time point *bcp* during period p in the aforementioned three scenarios can be expressed with three corresponding terms on the right-hand side of the following equation:

$$U_{i,j,p}^{L,bcp} = \sum_{k=0}^{p-1}\left[a_{i,j,k,p}^{L,bcp}\left(1 - X_{i,j,p}\right)Y_{i,j,k}\prod_{v=k+1}^{p}\left(1 - Y_{i,j,v}\right)\right] + bsp_{i,j,p}^{L,bcp}X_{i,j,p}Y_{i,j,p} + c_{i,j,p}^{L,bcp}(1 - X_{i,j,p})\prod_{z=0}^{p}\left(1 - Y_{i,j,z}\right) \tag{15}$$

where, $(i, j) \in$ E; $p = 1, 2, \cdots, n$; $a_{i,j,k,p}^{L,bcp}$ denotes the overall heat-transfer coefficient at time point *bcp* during period p ($p \geq 2$) in scenario (i) if exchanger (i, j) is last cleaned during period k ($1 \leq k < p$); $bsp_{i,j,p}^{L,bcp}$ is the overall heat-transfer coefficient at time point *bcp* during period p in scenario (ii) if a spare is adopted to replace exchanger (i, j); $c_{i,j,p}^{L,bcp}$ is the corresponding overall heat-transfer coefficient in scenario (iii). More specifically, the three coefficients mentioned above can be expressed explicitly as

$$a_{i,j,k,p}^{L,bcp} = \frac{1}{\frac{1}{\eta_{cl}U_{i,j}^{cl}} + \dot{r}_{i,j}\left[(p-k)\tau - f_c\right]} \tag{16}$$

$$bsp_{i,j,p}^{L,bcp} = \eta_{cl}U_{sp}^{cl} \tag{17}$$

$$c_{i,j,p}^{L,bcp} = \frac{1}{\frac{1}{U_{i,j}^{cl}} + \dot{r}_{i,j}(p-1)\tau} \tag{18}$$

Notice that in the above equations $a_{i,j,k,p}^{L,bcp}$ and $c_{i,j,p}^{L,bcp}$ vary with p and/or k, while $bsp_{i,j,p}^{L,bcp}$ is always a constant.

From equation (10), one can deduce that (a) $X_{i,j,p} = 0$ in the first and third scenarios due to $Y_{i,j,p} = 0$ and (b) $X_{i,j,p} \in \left\{0, 1\right\}$ in the second due to $Y_{i,j,p} = 1$. Note also that, when $p = 1$, the first term in equation (15) vanishes b$\eta_{cl}U_{sp}^{cl}$ecause of the permanent setting $Y_{i,j,0} = 0$. Thus, $U_{i,j,1}^{L,bcp}$ equals $U_{i,j}^{cl}$ if no cleaning take place in period 1 while this same coefficient may assume two alternative values, i.e., $\eta_{cl}U_{sp}^{cl}$ or 0, depending upon whether or not a spare is chosen to replace exchanger (i, j) to facilitate the defouling operation. In cases when $p \geq 2$, all three scenarios are possible and exactly one of corresponding terms in equation (15) remains after fixing the values of $Y_{i,j,p}$ and $X_{i,j,p}$.

After a time interval of length f_c, the overall heat-transfer coefficient of exchanger (i, j) at time point *ecp* during period p can be expressed by incorporating into equation (15) the extra fouling resistance increased since time point *bcp*, i.e.

$$U_{i,j,p}^{L,ecp} = \sum_{k=0}^{p-1}\left[a_{i,j,k,p}^{L,ecp}\left(1 - X_{i,j,p}\right)Y_{i,j,k}\prod_{v=k+1}^{p}\left(1 - Y_{i,j,v}\right)\right] + bsp_{i,j,p}^{L,ecp}X_{i,j,p}Y_{i,j,p} + c_{i,j,p}^{L,ecp}\left(1 - X_{i,j,p}\right)\prod_{z=0}^{p}\left(1 - Y_{i,j,z}\right) \tag{19}$$

where, $(i, j) \in$ E; $p = 1, 2, \cdots, n$; $a_{i,j,k,p}^{L,ecp}$ denotes the overall heat-transfer coefficient at time point *ecp* during period p ($p \geq 2$) in scenario (i) if exchanger (i, j) is cleaned during period k ($1 \leq k < p$) for the last time; $bsp_{i,j,p}^{L,ecp}$ is the overall heat-transfer coefficient at time point *ecp* during period p in scenario (ii) if a spare is adopted to replace exchanger (i, j); $c_{i,j,p}^{L,ecp}$ is the corresponding overall heat-transfer coefficient in scenario (iii). More specifically, the three coefficients mentioned above can be expressed explicitly as

$$a_{i,j,k,p}^{L,ecp} = \frac{1}{\frac{1}{\eta_{cl}U_{i,j}^{cl}} + (p-k)\dot{r}_{i,j}\tau} \tag{20}$$

$$bsp_{i,j,p}^{L,ecp} = \frac{1}{\frac{1}{\eta_{cl}U_{sp}^{cl}} + \dot{r}_{i,j}f_c} \tag{21}$$

$$c_{i,j,p}^{L,ecp} = \frac{1}{\frac{1}{U_{i,j}^{cl}} + \dot{r}_{i,j}\left[(p-1)\tau + f_c\right]} \tag{22}$$

Notice that $a_{i,j,k,p}^{L,ecp}$ and $c_{i,j,p}^{L,ecp}$ are also dependent upon p and/or k, but $bsp_{i,j,p}^{L,ecp}$ is a constant.

Note that, since exchanger (i, j) is not cleaned during period p in scenarios (i) and (iii), equations (20) and (22) should also be applicable for time point *bop* as well. Thus, the overall heat-transfer coefficient of exchanger (i, j) at instance *bop* during period p can be expressed as follows

$$U_{i,j,p}^{L,bop} = \sum_{k=0}^{p-1} \left[a_{i,j,k,p}^{L,bop} Y_{i,j,k} \prod_{v=k+1}^{p} \left(1 - Y_{i,j,v}\right) \right] + b_{i,j,p}^{L,bop} Y_{i,j,p} + c_{i,j,p}^{L,bop} \prod_{z=0}^{p} \left(1 - Y_{i,j,z}\right) \quad (23)$$

where, $(i, j) \in$ E; $p = 1, 2, \cdots, n$; $a_{i,j,k,p}^{L,bop} (= a_{i,j,k,p}^{L,ecp})$ and $c_{i,j,p}^{L,bop}$ $(= c_{i,j,p}^{L,ecp})$ have already been defined in equations (20) and (22) respectively; $b_{i,j,p}^{L,bop}$ denotes the overall heat-transfer coefficient at time point *bop* during period p in scenario (ii), i.e.

$$b_{i,j,p}^{L,bop} = \eta_c U_{i,j}^{cl} \quad (24)$$

Again $b_{i,j,p}^{L,bop}$ here is a constant.

Finally, the formulas for representing the overall heat-transfer coefficient of exchanger (i, j) at time point *eop* during period p can be obtained by introducing into equation (23) the additional fouling resistance increased since time point *bop*, i.e.

$$U_{i,j,p}^{L,eop} = \sum_{k=0}^{p-1} \left[a_{i,j,k,p}^{L,eop} Y_{i,j,k} \prod_{v=k+1}^{p} \left(1 - Y_{i,j,v}\right) \right] + b_{i,j,p}^{L,eop} Y_{i,j,p} + c_{i,j,p}^{L,eop} \prod_{z=0}^{p} \left(1 - Y_{i,j,z}\right) \quad (25)$$

where, $(i, j) \in$ E; $p = 1, 2, \cdots, n$; $a_{i,j,k,p}^{L,eop}$ denotes the overall heat-transfer coefficient at time point *eop* during period p $(p \geq 2)$ in scenario (i) if exchanger (i, j) is last cleaned during period k $(1 \leq k < p)$; $b_{i,j,p}^{L,eop}$ and $c_{i,j,p}^{L,eop}$ denote the overall heat-transfer coefficients at time point *eop* during period p in scenarios (ii) and (iii) respectively. The aforementioned three coefficients can be written explicitly as

$$a_{i,j,k,p}^{L,eop} = \frac{1}{\frac{1}{\eta_{cl} U_{i,j}^{cl}} + \dot{r}_{i,j} \left[(p - k + 1) \tau - f_c \right]} \quad (26)$$

$$b_{i,j,p}^{L,eop} = \frac{1}{\frac{1}{\eta_{cl} U_{i,j}^{cl}} + \dot{r}_{i,j} (\tau - f_c)} \quad (27)$$

$$c_{i,j,p}^{L,eop} = \frac{1}{\frac{1}{U_{i,j}^{cl}} + \dot{r}_{i,j} p \tau} \quad (28)$$

Note that $a_{i,j,k,p}^{L,eop}$ and $c_{i,j,p}^{L,eop}$ are both variables depending upon p and/or k, and $b_{i,j,p}^{L,eop}$ is a constant.

7.2. Exchanger inlet and outlet temperatures

By assuming pseudo counter-current flow in every heat exchanger, the corresponding energy balance can be written as

$$\begin{aligned} Q_{i,j}(t) &= F_i^H C_i^H \left(T_{in,i}^H(t) - T_{out,i}^H(t)\right) = F_j^C C_j^C \left(T_{out,j}^C(t) - T_{in,j}^C(t)\right) \\ &= U_{i,j}(t) A_{i,j} \frac{\left(T_{in,i}^H(t) - T_{out,j}^C(t)\right) - \left(T_{out,i}^H(t) - T_{in,j}^C(t)\right)}{\ln\left[\left(T_{in,i}^H(t) - T_{out,j}^C(t)\right) / \left(T_{out,i}^H(t) - T_{in,j}^C(t)\right)\right]} \end{aligned} \quad (29)$$

where, $(i, j) \in$ E; $t \in [0, t_f]$; $Q_{i,j}$ is the heat duty (kJ/s) of exchanger (i, j); $A_{i,j}$ is the heat transfer area (m^2) of exchanger (i, j) and it is a given model parameter; $F_i^H$and F_j^C denote respectively the mass flow rates (kg/s) of hot stream i and cold stream j and they are model parameters; $C_i^H$and C_j^C denote respectively the heat capacities (kJ/kg-K) of hot stream i and cold stream j and they are also model parameters; $T_{in,i}^H$and $T_{in,j}^C$ denote respectively the inlet temperatures (K) of hot stream i and cold stream j; $T_{out,i}^H$and $T_{out,j}^C$ denote respectively the outlet temperatures (K) of hot stream i and cold stream j. Equation (29) can then be rearranged to produce an expression for the outlet temperature of the hot stream, i.e.

$$T_{out,i}^H(t) = \frac{\left(R_{i,j} - 1\right) T_{in,i}^H(t) + \left\{ \exp\left[\frac{U_{i,j}(t) A_{i,j}}{F_j^C C_j^C}\left(R_{i,j} - 1\right)\right] - 1 \right\} R_{i,j} T_{in,j}^C(t)}{R_{i,j} \exp\left[\frac{U_{i,j}(t) A_{i,j}}{F_j^C C_j^C}\left(R_{i,j} - 1\right)\right] - 1} \quad (30)$$

where, the constant $R_{i,j}$ is defined as

$$R_{i,j} = \frac{F_j^C C_j^C}{F_i^H C_i^H} = \frac{T_{out,i}^H(t) - T_{in,i}^H(t)}{T_{in,j}^C(t) - T_{out,j}^C(t)} \quad (31)$$

To simplify model formulation, let us introduce an additional model parameter $d_{i,j}$ and also a variable $d_{i,j}^{sp}$:

$$d_{i,j} = \frac{A_{i,j}}{F_j^C C_j^C}\left(R_{i,j} - 1\right) \quad (32)$$

$$d_{i,j}^{sp} = \frac{A_{sp}}{F_j^C C_j^C}\left(R_{i,j} - 1\right) \quad (33)$$

where, A_{sp} is the heat-transfer area of the spare exchanger and it is treated as a *variable* in the mathematical programming model. Note that, to facilitate time sharing of the spares, it is assumed in this work that their heat-transfer areas are chosen to be identical.

After substituting equations (15), (19), (23) and (25) into equation (30), one can then obtain the following formulas to determine the outlet hot stream temperatures at the aforementioned four time points:

$$\begin{aligned}
T_{out,i,p}^{H,bcp} = {} & \\
& \left(1 - X_{i,j,p}\right) Y_{i,j,p} T_{in,i,p}^{H,bcp} \\
& + X_{i,j,p} Y_{i,j,p} \left\{ \frac{Y_{i,j,p}}{R_{i,j}\exp\left(d_{i,j}^{sp} bsp_{i,j,p}^{L,bcp}\right) - 1}\left[\left(R_{i,j} - 1\right) T_{in,i,p}^{H,bcp} - R_{i,j} T_{in,j,p}^{C,bcp}\right] + R_{i,j} T_{in,j,p}^{C,L,bcp} \frac{Y_{i,j,p}\exp\left(d_{i,j}^{sp} bsp_{i,j,p}^{L,bcp}\right)}{R_{i,j}\exp\left(d_{i,j}^{sp} bsp_{i,j,p}^{L,bcp}\right) - 1} \right\} \\
& + \left(1 - X_{i,j,p}\right)\left(1 - Y_{i,j,p}\right) \left\{ \left[\left(R_{i,j} - 1\right) T_{in,i,p}^{H,bcp} - R_{i,j} T_{in,j,p}^{C,bcp}\right] \left[\sum_{k=0}^{p-1} \frac{Y_{i,j,k} \prod_{v=k+1}^{p} \left(1 - Y_{i,j,v}\right)}{R_{i,j}\exp\left(d_{i,j} a_{i,j,k,p}^{L,bcp}\right) - 1} + \frac{\prod_{z=0}^{p}\left(1 - Y_{i,j,z}\right)}{R_{i,j}\exp\left(d_{i,j} c_{i,j,p}^{L,bcp}\right) - 1} \right] \right. \\
& \left. + R_{i,j} T_{in,j,p}^{C,bcp} \left[\sum_{k=0}^{p-1} \frac{Y_{i,j,k} \prod_{v=k+1}^{p} \left(1 - Y_{i,j,v}\right)\exp\left(d_{i,j} a_{i,j,k,p}^{L,bcp}\right)}{R_{i,j}\exp\left(d_{i,j} a_{i,j,k,p}^{L,bcp}\right) - 1} + \frac{\prod_{z=0}^{p}\left(1 - Y_{i,j,z}\right)\exp\left(d_{i,j} c_{i,j,p}^{L,bcp}\right)}{R_{i,j}\exp\left(d_{i,j} c_{i,j,p}^{L,bcp}\right) - 1} \right] \right\}
\end{aligned} \quad (34)$$

$$\begin{aligned}
T_{out,i,p}^{H,ecp} = {} & \\
& \left(1 - X_{i,j,p}\right) Y_{i,j,p} T_{in,i,p}^{H,ecp} \\
& + X_{i,j,p} Y_{i,j,p} \left\{ \frac{Y_{i,j,p}}{R_{i,j}\exp\left(d_{i,j}^{sp} bsp_{i,j,p}^{L,ecp}\right) - 1}\left[\left(R_{i,j} - 1\right) T_{in,i,p}^{H,ecp} - R_{i,j} T_{in,j,p}^{C,ecp}\right] + R_{i,j} T_{in,j,p}^{C,ecp} \frac{Y_{i,j,p}\exp\left(d_{i,j}^{sp} bsp_{i,j,p}^{L,ecp}\right)}{R_{i,j}\exp\left(d_{i,j}^{sp} bsp_{i,j,p}^{L,ecp}\right) - 1} \right\} \\
& + \left(1 - X_{i,j,p}\right)\left(1 - Y_{i,j,p}\right) \left\{ \left[\left(R_{i,j} - 1\right) T_{in,i,p}^{H,ecp} - R_{i,j} T_{in,j,p}^{C,ecp}\right] \left[\sum_{k=0}^{p-1} \frac{Y_{i,j,k} \prod_{v=k+1}^{p} \left(1 - Y_{i,j,v}\right)}{R_{i,j}\exp\left(d_{i,j} a_{i,j,k,p}^{L,ecp}\right) - 1} + \frac{\prod_{z=0}^{p}\left(1 - Y_{i,j,z}\right)}{R_{i,j}\exp\left(d_{i,j} c_{i,j,p}^{L,ecp}\right) - 1} \right] \right. \\
& \left. + R_{i,j} T_{in,j,p}^{C,ecp} \left[\sum_{k=0}^{p-1} \frac{Y_{i,j,k} \prod_{v=k+1}^{p} \left(1 - Y_{i,j,v}\right)\exp\left(d_{i,j} a_{i,j,k,p}^{L,ecp}\right)}{R_{i,j}\exp\left(d_{i,j} a_{i,j,k,p}^{L,ecp}\right) - 1} + \frac{\prod_{z=0}^{p}\left(1 - Y_{i,j,z}\right)\exp\left(d_{i,j} c_{i,j,p}^{L,ecp}\right)}{R_{i,j}\exp\left(d_{i,j} c_{i,j,p}^{L,ecp}\right) - 1} \right] \right\}
\end{aligned} \quad (35)$$

$$T_{out,i,p}^{H,bop} = \\ \left[\left(R_{i,j}-1\right)T_{in,i,p}^{H,bop} - R_{i,j}T_{in,j,p}^{C,bop}\right]\left[\sum_{k=0}^{p-1}\frac{Y_{i,j,k}\prod_{v=k+1}^{p}\left(1-Y_{i,j,v}\right)}{R_{i,j}\exp\left(d_{i,j}a_{i,j,k,p}^{L,bop}\right)-1} + \frac{Y_{i,j,p}}{R_{i,j}\exp\left(d_{i,j}b_{i,j,p}^{L,bop}\right)-1} + \frac{\prod_{z=0}^{p}\left(1-Y_{i,j,z}\right)}{R_{i,j}\exp\left(d_{i,j}c_{i,j,p}^{L,bop}\right)-1}\right] \\ +R_{i,j}T_{in,j,p}^{C,bop}\left[\sum_{k=0}^{p-1}\frac{Y_{i,j,k}\prod_{v=k+1}^{p}\left(1-Y_{i,j,v}\right)\exp\left(d_{i,j}a_{i,j,k,p}^{L,bop}\right)}{R_{i,j}\exp\left(d_{i,j}a_{i,j,k,p}^{L,bop}\right)-1} + \frac{Y_{i,j,p}\exp\left(d_{i,j}b_{i,j,p}^{L,bop}\right)}{R_{i,j}\exp\left(d_{i,j}b_{i,j,p}^{L,bop}\right)-1} + \frac{\prod_{z=0}^{p}\left(1-Y_{i,j,z}\right)\exp\left(d_{i,j}c_{i,j,p}^{L,bop}\right)}{R_{i,j}\exp\left(d_{i,j}c_{i,j,p}^{L,bop}\right)-1}\right] \tag{36}$$

$$T_{out,i,p}^{H,eop} = \\ \left[\left(R_{i,j}-1\right)T_{in,i,p}^{H,eop} - R_{i,j}T_{in,j,p}^{C,eop}\right]\left[\sum_{k=0}^{p-1}\frac{Y_{i,j,k}\prod_{v=k+1}^{p}\left(1-Y_{i,j,v}\right)}{R_{i,j}\exp\left(d_{i,j}a_{i,j,k,p}^{L,eop}\right)-1} + \frac{Y_{i,j,p}}{R_{i,j}\exp\left(d_{i,j}b_{i,j,p}^{L,eop}\right)-1} + \frac{\prod_{z=0}^{p}\left(1-Y_{i,j,z}\right)}{R_{i,j}\exp\left(d_{i,j}c_{i,j,p}^{L,eop}\right)-1}\right] \\ +R_{i,j}T_{in,j,p}^{C,eop}\left[\sum_{k=0}^{p-1}\frac{Y_{i,j,k}\prod_{v=k+1}^{p}\left(1-Y_{i,j,v}\right)\exp\left(d_{i,j}a_{i,j,k,p}^{L,eop}\right)}{R_{i,j}\exp\left(d_{i,j}a_{i,j,k,p}^{L,eop}\right)-1} + \frac{Y_{i,j,p}\exp\left(d_{i,j}b_{i,j,p}^{L,eop}\right)}{R_{i,j}\exp\left(d_{i,j}b_{i,j,p}^{L,eop}\right)-1} + \frac{\prod_{z=0}^{p}\left(1-Y_{i,j,z}\right)\exp\left(d_{i,j}c_{i,j,p}^{L,eop}\right)}{R_{i,j}\exp\left(d_{i,j}c_{i,j,p}^{L,eop}\right)-1}\Bigg\}\right] \tag{37}$$

where, $(i, j) \in$ E; $p = 1, 2, \cdots, n$; $T_{out,i,p}^{H,tp}$ denotes the outlet temperature of hot stream i at time point $tp \in \left\{bcp, ecp, bop, eop\right\}$ in period p; $T_{in,i,p}^{H,tp}$ and $T_{in,j,p}^{C,tp}$ denote respectively the inlet temperatures of hot stream i and clod stream j at time point $tp \in \left\{bcp, ecp, bop, eop\right\}$ in period p.

Finally, the outlet temperatures of cold stream at different time instances can be determined according to equation (31), i.e.

$$T_{out,j,p}^{C,tp} = T_{in,j,p}^{C,tp} + \frac{T_{in,i,p}^{H,tp} - T_{out,i,p}^{H,tp}}{R_{i,j}} \tag{38}$$

where, $(i, j) \in$ E; $p = 1, 2, \cdots, n$; $tp \in \left\{bcp, ecp, bop, eop\right\}$; $T_{out,j,p}^{C,tp}$ denotes the outlet temperature of cold stream j at time point tp in period p.

7.3. Utility consumption rates

Since the original HEN design specifications are given, a straightforward computation can be performed to determine the utility consumption rate needed to bring the final temperature of each process stream to its target value.

$$Qu_{j,p}^{H,tp} = F_j^C C_j^C \left(TT_j^C - TL_{j,p}^{C,tp}\right) \;\; \forall j \in \mathrm{J} \tag{39}$$

$$Qu_{i,p}^{C,tp} = F_i^H C_i^H \left(TL_{i,p}^{H,tp} - TT_i^H\right) \;\; \forall i \in \mathrm{I} \tag{40}$$

where, $tp \in \left\{bcp, ecp, bop, eop\right\}$; TT_j^C and TT_i^H denote the target temperatures of cold stream j and hot steam i respectively; $TL_{j,p}^{C,tp}$ and $TL_{i,p}^{H,tp}$ denote the outlet temperatures of the last unit on cold stream j and hot steam i respectively at time point tp; $Qu_{j,p}^{H,tp}$ is the hot utility consumption rate needed by cold stream j at time point tp; $Qu_{i,p}^{C,tp}$ is the cold utility consumption rate needed by hot stream i at time point tp. Consequently, the total amounts of utilities consumed respectively by cold stream j and hot stream i in period p can be estimated according to the following formulas:

$$Eu_{j,p}^{H} = \frac{Qu_{j,p}^{H,bcp} + Qu_{j,p}^{H,ecp}}{2} f_c + \frac{Qu_{j,p}^{H,bop} + Qu_{j,p}^{H,eop}}{2}\left(\tau - f_c\right) \;\; j \in \mathrm{J} \tag{41}$$

$$Eu_{i,p}^{C} = \frac{Qu_{i,p}^{C,bcp} + Qu_{i,p}^{C,ecp}}{2} f_c + \frac{Qu_{i,p}^{C,bop} + Qu_{i,p}^{C,eop}}{2}\left(\tau - f_c\right) \;\; i \in \mathrm{I} \tag{42}$$

8. Objective function – total annual cost

As mentioned before, the maximum length of time horizon allowed for schedule synthesis is chosen to be the duration between the ending and beginning instances of two consecutive planned shutdowns, i.e. t_f. The total annual cost (TAC) associated with operating and

Table 3
Cost estimates obtained under linear fouling assumption in Example 1 (million USD/year).

	No cleaning for 1 year	Regular cleaning for 1 year
Cold utility	0.75	0.74
Hot utility	1.91	1.87
Cleaning	0.00	0.008
TAOC	2.660	2.618

cleaning a given HEN can be approximated by summing the total annualized capital cost (TACC) for purchasing the spares and the average value of the total annual operating cost ($\overline{\text{TAOC}}$), i.e.

$$\text{TAC} = \text{TACC} + \overline{\text{TAOC}} \tag{43}$$

The former cost is computed with the following formula

$$\text{TACC} = N_{sp} C_{sp} A_{sp}^{0.8} \tag{44}$$

On the other hand, the latter is approximated simply by taking the arithmetic average of total operating costs needed in all years in the schedule horizon, i.e.

$$\overline{\text{TAOC}} = \frac{1}{I_{tf}} \sum_{k=1}^{I_{tf}} \text{TOC}_k \tag{45}$$

where, TOC_k denotes the total operating cost in year k, which can be further divided into the total utility cost (TUC_k) and the total cleaning cost (TCLC_k). Specifically,

$$\text{TOC}_k = \text{TUC}_k + \text{TCLC}_k \tag{46}$$

$$\text{TUC}_k = \sum_{p \in \text{P}_k} \left[\frac{C_p^{CU}}{\eta^C} \sum_{i \in \text{I}} Eu_{i,p}^C + \frac{C_p^{HU}}{\eta^H} \sum_{j \in \text{J}} Eu_{j,p}^H \right] \tag{47}$$

$$\text{TCLC}_k = C_{cl} \sum_{p \in \text{P}_k} \sum_{(i,j) \in \text{E}} Y_{i,j,p} + C_{cl}^{sp} \sum_{p \in \text{P}_k} \sum_{(i,j) \in \text{E}} X_{i,j,p} \tag{48}$$

where, C_p^{CU} and C_p^{HU} denote the unit costs of cooling and heating utilities in period p respectively; η^C and η^H denote the heat-transfer efficiencies in cooler and heater respectively; C_{cl} and C_{cl}^{sp} represent the costs of cleaning a heat exchanger and a spare respectively.

9. Case studies

All aforementioned model constraints have been utilized to construct the MINLP models for synthesizing the spare-supported HEN cleaning schedules in two examples. The first is adopted primarily to show the potential benefit of cleaning. The second is used to compare the effects of two intrinsic parameters, i.e., the schedule horizons and the fouling rates (which are characterized respectively with the linear and exponential models). Starting from randomly generated initial guesses, these models were solved repeatedly (say 20 runs) with solver DICOPT in GAMS 23.9.5 so as to ensure solution quality. Finally, it should be noted that all such computations were carried out on a PC equipped with Intel core i7-4770 3.4 GHz.

9.1. Example 1

Let us first consider the HEN presented in Fig. 1, the corresponding stream data in Table 1 and the design specifications in Table 2. In this example, linear fouling is assumed and its rate $\dot{r}_{i,j}$ in every exchanger is set at 2.2×10^{-8} ($\text{m}^2\,\text{K}\,\text{kW}^{-1}\,\text{mon}^{-1}$). The unit costs of hot and cold utilities are chosen to be 4×10^{-6} USD/kJ and 4×10^{-7} USD/kJ respectively, while the cleaning cost of either an online exchanger or a spare is fixed at 4000 USD. The other model parameters are selected as follows: $t_f = 12$ (mon); $\tau = 1$ (mon); $f_c = 0.2$ (mon); $\eta^{cl} = 0.75$; $\eta^H = 1$; $\eta^C = 1$; $C_{sp} = 1000$(USD $\text{yr}^{-1}\,\text{m}^{-1.6}$). By solving the proposed mathematical programming model, it was found that the spares are not needed at all. The resulting optimal schedule calls for respectively cleaning HE3 and HE4 during the 7th and 6th month. From the cost summary presented in Table 3, it can be observed that a slight reduction in TAOC can be achieved by cleaning the heat exchangers. Although in the present case only marginal improvement in the operating cost can be realized, it should be noted that more pronounced cost savings can be achieved with either regular or spare-supported cleaning strategies in a larger system.

9.2. Example 2

This example is adopted from a HEN synthesis problem studied in Papoulias and Grossmann (1983). The flow diagram of HEN is presented in Fig. 3, while the corresponding stream data and design specifications can be found in Tables 4 and 5 respectively. The unit costs of hot and cold utilities are chosen to be 4×10^{-6} USD/kJ and 4×10^{-7} USD/kJ respectively, while the cleaning cost of either an online exchanger or a spare is fixed at 4000 USD. The other model parameters are selected as follows: $\tau = 1$ mon; $f_c = 0.2$ mon; $\eta^{cl} = 0.75$; $\eta^H = 1$; $\eta^C = 1$; $C_{sp} = 1000$(USD $\text{yr}^{-1}\,\text{m}^{-1.6}$). Both linear and exponential fouling models are adopted for schedule synthesis in the present example. The linear fouling rate $\dot{r}_{i,j}$ in every exchanger is set at 0.057 ($\text{m}^2\,\text{K}\,\text{kW}^{-1}\,\text{mon}^{-1}$), while the asymptotic maximum fouling resistance $r_{i,j}^{\infty}$ and the

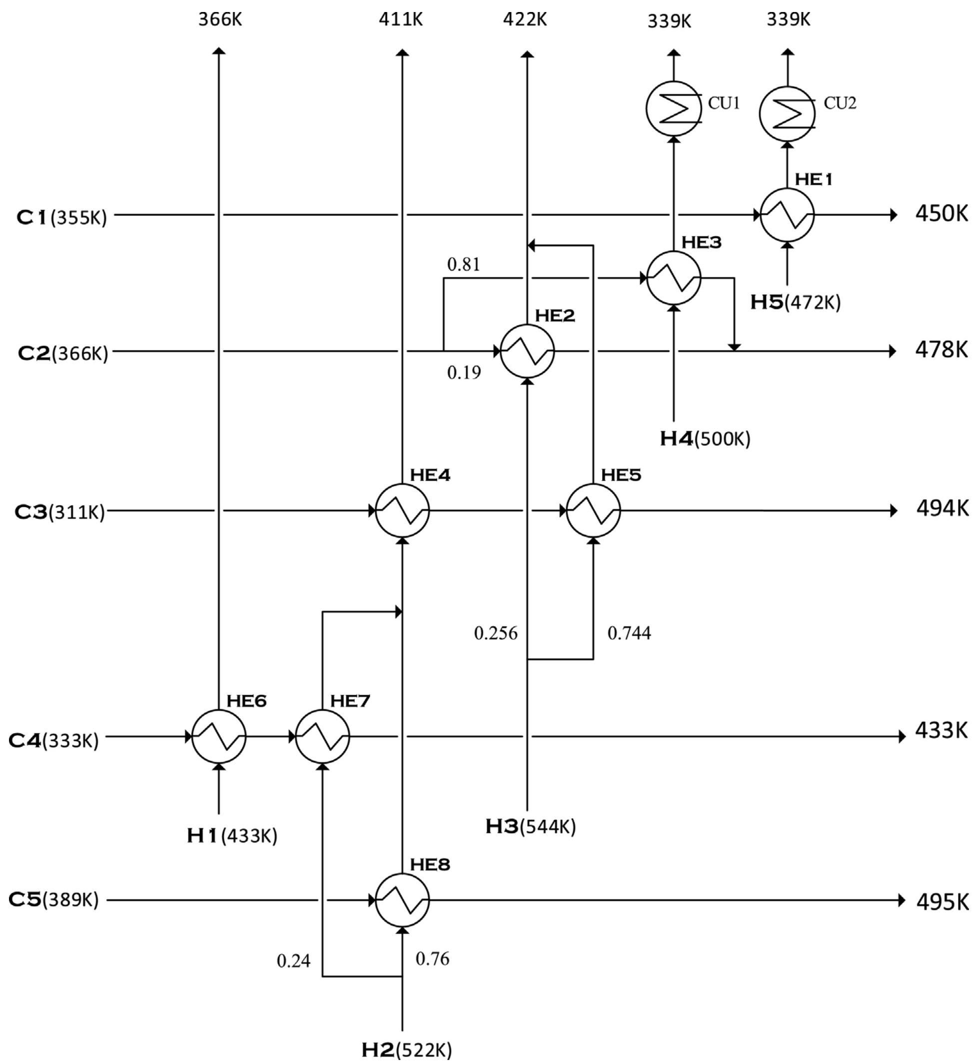

Fig. 3. Flow diagram of HEN considered in Example 2.

Table 4
Stream data of Example 2.

Stream	Initial Temp. (K)	Target Temp. (K)	Heat Capacity Flow Rate (kW/K)
H1	433	366	8.79
H2	522	411	10.55
H3	544	422	12.56
H4	500	339	14.77
H5	472	339	17.73
C1	355	450	17.28
C2	366	478	13.90
C3	311	494	8.44
C4	333	453	7.62
C5	359	495	6.08

Table 5
Design specifications of heat exchangers in Example 2.

Unit	$U_{i,j}^{cl}$(kW/m^2 K)	$A_{i,j}$(m^2)	Cold Stream Inlet and Outlet Temp. (K)	Hot Stream Inlet and Outlet Temp. (K)	Heat Duty (kW)
HE1	0.85	82.94	355/450	472/380	1641.6
HE2	0.85	6.7	366/495	544/438	341.3
HE3	0.85	38.2	366/474	500/418	1215.5
HE4	0.85	4.33	311/353	445/411	173.1
HE5	0.85	24.7	353/494	544/417	1191
HE6	0.85	25.1	333/410	433/366	588.9
HE7	0.85	3.26	410/433	522/453	644.5
HE8	0.85	19.57	389/495	522/442	644.5
CU1	0.85	32.68	311/355	418/339	1170.8
CU2	0.85	33.16	311/355	380/339	718.2

Table 6
Regular 1-year cleaning schedule in Example 2 (linear fouling).

Unit	Month										
	1	2	3	4	5	6	7	8	9	10	11	12
HE1							○					
HE2							○					
HE3								○				
HE4							○					
HE5						○						
HE6							○					
HE7						○						
HE8						○						

○: regular cleaning operation.

Table 7
Spare-supported 1-year cleaning schedule in Example 2 (linear fouling).

Unit	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
HE1							●					
HE2					●			●				
HE3								●				
HE4						●						
HE5							●					
HE6					●							
HE7						●						
HE8							○					

○: regular cleaning operation; ●: spare-supported cleaning operation.

Table 8
Spare-supported 2-year cleaning schedule in Example 2 (linear fouling).

Unit	Month															
	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
HE1	●						●					●				
HE2	●							●								●
HE3			●						●					●		
HE4	○						○						●			
HE5		●					●			●					●	
HE6			●						●				●			
HE7		○						●						●		
HE8		●								●					●	

○: regular cleaning operation; ●: spare-supported cleaning operation.

Table 9
Cost estimates obtained under linear fouling assumption in Example 2 (million USD/year).

	No cleaning for 1 year	No cleaning for 2 year	Regular cleaning for 1 year	Spare-supported cleaning for 1 year	Spare-supported cleaning for 2 years
Cold U.	0.28	0.33	0.27	0.25	0.26
Hot U.	0.52	0.95	0.39	0.27	0.24
Cleaning	0.00	0.00	0.03	0.07	0.09
Spares	0.00	0.00	0.00	0.04	0.05
TAC	0.80	1.28	0.69	0.63	0.64

characteristic fouling speed $K_{i,j}$ in the exponential model are chosen to be 0.684 (m^2 K/kW) and 0.25 (mon^{-1}) respectively for each unit in HEN. Since in this case $r_{i,j}^{\infty} = 12\dot{r}_{i,j}$, the short-term fouling rate characterized by the exponential model should be larger than that by the linear model. For illustration clarity, let us present and discuss the cleaning schedules generated on the basis of these two different models separately in sequence:

- Tables 6 and 7 respectively shows the regular and spare-supported 1-year cleaning schedules obtained by solving the proposed mathematical programming models under the assumption of linear fouling. In the latter case, it was determined that two identical spares are needed and each has a heat-transfer area of 35.95 m^2. Table 8 shows the spare-supported cleaning schedule for an operation horizon of 2 years. For this case, the computation time of each optimization run is approximately 9 s. Notice also that the number of spares needed to implement this schedule is also 2 and they both have the same area of 51.61 m^2. From the cost estimates of the 1-year cleaning schedules presented in Table 9, it can be observed that roughly a saving of 18.8% TAC can be achieved with regular cleaning and a further reduction of 7.1% TAC if spares are adopted. However, by comparing the TACs of the 1-year and 2-year spare-supported cleaning schedules, one can see that there are essentially no benefits to extend the horizon to a longer period.
- Tables 10 and 11 respectively shows the regular and spare-supported 1-year cleaning schedules obtained by solving the proposed mathematical programming models under the assumption of exponential fouling. The latter schedule only calls for one spare with a heat-transfer area of 69.14 m^2. The optimal spare-supported 2-year cleaning schedule can be found in Table 12, and it took 40 s to

Table 10
Regular 1-year cleaning schedule in Example 2 (exponential fouling).

Unit	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
HE1						○						
HE2						○						
HE3								○				
HE4						○						
HE5					○				○			
HE6								○				
HE7					○				○			
HE8					○				○			

○: regular cleaning operation.

Table 11
Spare-supported 1-year cleaning schedule in Example 2 (exponential fouling).

Unit	Month											
	1	2	3	4	5	6	7	8	9	10	11	12
HE1							●					
HE2							○					
HE3						●				●		
HE4					●				●			
HE5					○				○			
HE6								●				
HE7					○				○			
HE8					○				○			

○: regular cleaning operation; ●: spare-supported cleaning operation.

Table 12
Spare-supported 2-year cleaning schedule in Example 2 (exponential fouling).

Unit	Month																	
	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
HE1			●					●					●					●
HE2		●				●					●					●		
HE3			●					●				●			●			
HE4				●							●					●		
HE5	●				●		●			●		●				○	●	
HE6				●					●						●			
HE7				○													●	
HE8		●			●				●					●				●

○: regular cleaning operation; ●: spare-supported cleaning operation.

Table 13
Cost estimates obtained under exponential fouling assumption in Example 2 (million USD/year).

	No cleaning for 1 year	No cleaning for 2 year	Regular cleaning for 1 year	Spare-supported cleaning for 1 year	Spare-supported cleaning for 2 years
Cold U.	0.31	0.32	0.30	0.28	0.27
Hot U.	0.76	0.875	0.67	0.51	0.39
Cleaning	0.00	0.00	0.05	0.08	0.12
Spares	0.00	0.00	0.00	0.03	0.06
TAC	1.07	1.195	1.02	0.90	0.84

complete each repeated optimization run. Notice also that the number of spares needed to implement this schedule is increased to 2 and they both have the same area of 70.21 m^2. From the cost estimates of the 1-year cleaning schedules presented in Table 13, it can be observed that roughly a saving of 8.9% TAC can be achieved with regular cleaning and an additional reduction of another 8.9% TAC can also be achieved with spares. Furthermore, by comparing the TACs of the spare-supported cleaning schedules, one can see that an extra 6.7% decrease can be realized by extending the operation horizon from 1 to 2 years.

10. Conclusions

An improved mathematical programming model has been developed in this work to synthesize the optimal spare-supported cleaning schedule for any given heat exchanger network. The effectiveness of this approach has been verified with extensive case studies. It can be observed from the optimization results that spares are viable options for reducing the extra amount of utility consumption caused by temporarily removing online heat exchangers for cleaning purpose. In addition, potential financial benefit (in terms of TAC) may also be realized by extending the schedule horizon to a longer period of time. These operation strategies are especially effective when the short-term fouling rate is relatively large and thus cleaning is required more frequently.

Appendix A.

Model constraints under exponential fouling assumption

The linear-fouling based formulations in section 7 can be easily converted to a second set of model constraints under the exponential fouling assumption, i.e. equation (14). Specifically, by replacing the superscript L with E, the overall heat transfer coefficients can be rewritten as

$$U_{i,j,p}^{E,bcp} = \sum_{k=0}^{p-1}\left[a_{i,j,k,p}^{E,bcp}\left(1-X_{i,j,p}\right)Y_{i,j,k}\prod_{v=k+1}^{p}\left(1-Y_{i,j,v}\right)\right] + bsp_{i,j,p}^{E,bcp}X_{i,j,p}Y_{i,j,p} + c_{i,j,p}^{E,bcp}\left(1-X_{i,j,p}\right)\prod_{z=0}^{p}\left(1-Y_{i,j,z}\right) \quad \text{(A1)}$$

$$U_{i,j,p}^{E,ecp} = \sum_{k=0}^{p-1}\left[a_{i,j,k,p}^{E,ecp}\left(1-X_{i,j,p}\right)Y_{i,j,k}\prod_{v=k+1}^{p}\left(1-Y_{i,j,v}\right)\right] + bsp_{i,j,p}^{E,ecp}X_{i,j,p}Y_{i,j,p} + c_{i,j,p}^{E,ecp}\left(1-X_{i,j,p}\right)\prod_{z=0}^{p}\left(1-Y_{i,j,z}\right) \quad \text{(A2)}$$

$$U_{i,j,p}^{E,bop} = \sum_{k=0}^{p-1}\left[a_{i,j,k,p}^{E,bop}Y_{i,j,k}\prod_{v=k+1}^{p}\left(1-Y_{i,j,v}\right)\right] + b_{i,j,p}^{E,bop}Y_{i,j,p} + c_{i,j,p}^{E,bop}\prod_{z=0}^{p}\left(1-Y_{i,j,z}\right) \quad \text{(A3)}$$

$$U_{i,j,p}^{E,eop} = \sum_{k=0}^{p-1}\left[a_{i,j,k,p}^{E,eop}Y_{i,j,k}\prod_{v=k+1}^{p}\left(1-Y_{i,j,v}\right)\right] + b_{i,j,p}^{E,eop}Y_{i,j,p} + c_{i,j,p}^{E,eop}\prod_{z=0}^{p}\left(1-Y_{i,j,z}\right) \quad \text{(A4)}$$

where,

$$a_{i,j,k,p}^{E,bcp} = \frac{1}{\frac{1}{\eta_{cl}U_{i,j}^{cl}} + r_{i,j}^{\infty}\{1-\exp[-K_{i,j}((p-k)\tau - f_c)]\}} \quad \text{(A5)}$$

$$bsp_{i,j,p}^{E,bcp} = \eta_{cl}U_{sp}^{cl} \quad \text{(A6)}$$

$$c_{i,j,p}^{E,bcp} = \frac{1}{\frac{1}{U_{i,j}^{cl}} + r_{i,j}^{\infty}\{1-\exp[-K_{i,j}(p-1)\tau]\}} \quad \text{(A7)}$$

$$a_{i,j,k,p}^{E,ecp} = \frac{1}{\frac{1}{\eta_{cl}U_{i,j}^{cl}} + r_{i,j}^{\infty}\{1-\exp[-K_{i,j}(p-k)\tau]\}} \quad \text{(A8)}$$

$$bsp_{i,j,p}^{E,ecp} = \frac{1}{\frac{1}{\eta_{cl}U_{sp}^{cl}} + r_{i,j}^{\infty}[1-\exp(-K_{i,j}f_c)]} \quad \text{(A9)}$$

$$c_{i,j,p}^{E,ecp} = \frac{1}{\frac{1}{U_{i,j}^{cl}} + r_{i,j}^{\infty}\{1-\exp[-K_{i,j}((p-1)\tau + f_c)]\}} \quad \text{(A10)}$$

$$a_{i,j,k,p}^{E,bop} = a_{i,j,k,p}^{E,ecp} = \frac{1}{\frac{1}{\eta_{cl}U_{i,j}^{cl}} + r_{i,j}^{\infty}\{1-\exp[-K_{i,j}(p-k)\tau]\}} \quad \text{(A11)}$$

$$b_{i,j,p}^{E,bop} = \eta_c U_{i,j}^{cl} \quad \text{(A12)}$$

$$c_{i,j,p}^{E,bop} = c_{i,j,p}^{E,ecp} = \frac{1}{\frac{1}{U_{i,j}^{cl}} + r_{i,j}^{\infty}\{1-\exp[-K_{i,j}((p-1)\tau + f_c)]\}} \quad \text{(A13)}$$

$$a_{i,j,k,p}^{E,eop} = \frac{1}{\frac{1}{\eta_{cl}U_{i,j}^{cl}} + r_{i,j}^{\infty}\{1-\exp[-K_{i,j}((p-k+1)\tau - f_c)]\}} \quad \text{(A14)}$$

$$b_{i,j,p}^{E,eop} = \frac{1}{\frac{1}{\eta_{cl}U_{i,j}^{cl}} + r_{i,j}^{\infty}\{1-\exp[-K_{i,j}(\tau - f_c)]\}} \quad \text{(A15)}$$

$$c^{E,eop}_{i,j,p} = \frac{1}{\frac{1}{U^{cl}_{i,j}} + r^{\infty}_{i,j}[1 - \exp(-K_{i,j}p\tau)]} \quad \text{(A16)}$$

It should be noted that in the above 12 equations $a^{E,bcp}_{i,j,k,p}$,$a^{E,ecp}_{i,j,k,p}$,$a^{E,bop}_{i,j,k,p}$,$a^{E,eop}_{i,j,k,p}$,$c^{E,bcp}_{i,j,p}$,$c^{E,ecp}_{i,j,p}$,$c^{E,bop}_{i,j,p}$ and $c^{E,eop}_{i,j,p}$ are all variables depending upon p and/or k, but $bsp^{E,bcp}_{i,j,p}$,$bsp^{E,ecp}_{i,j,p}$,$b^{E,bop}_{i,j,p}$ and $b^{E,eop}_{i,j,p}$ are constants. Furthermore, equations (34)–(37) should also be modified by replacing the superscript L with E. The corresponding hot stream temperatures at the outlet of each heat exchanger can therefore be determined according to equations (A5)–(A16).

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