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Optimal Retrofit of a Multiplant Heat Exchanger Network with a Fair Benefit Allocation Plan

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Cite This: *Ind. Eng. Chem. Res.* 2020, 59, 18088−18105 Read Online

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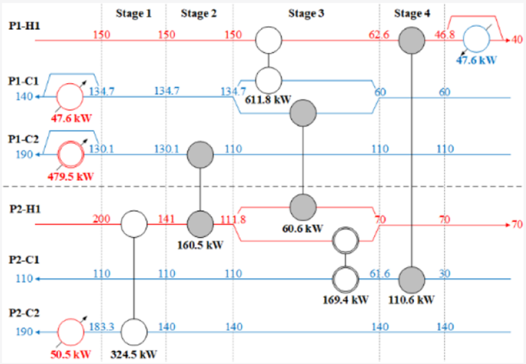

ABSTRACT: A model-based procedure has recently been developed to address the benefit allocation issue among members of an interplant heat integration scheme in the spirit of co-operative game. Although satisfactory results were reported, their approach is only applicable to grass-root designs. In practical applications, the existing plants on an industrial park were usually built to meet the targeted market demands, which arose during different periods in the past and each must have already been equipped with a heat exchanger network (HEN) by the time when its construction was completed. Therefore, the aforementioned allocation problem is more likely to take place when a multiplant HEN retrofit project is called for to facilitate a further reduction in utility consumption. This paper presents three viable strategies to solve the more realistic revamp problems. Depending upon other requirements in practical applications, for example, safety concerns, spatial limits, operability, and so forth, these strategies are devised by introducing different levels of restrictions on the new and original matches, on repiping and reusing of existing units in the multiplant HEN, and on the installation of purchased new heat exchangers, coolers, and heaters. The actual financial benefits allocated to the participating members of the interplant heat integration scheme are then determined according to the corresponding Shapley values. A simple example is presented in this paper to illustrate the aforementioned HEN retrofit and saving distribution procedures.

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1. INTRODUCTION

Heat exchanger network (HEN) is traditionally used for maximum heat recovery in a single chemical plant, while the multiplant counterparts have been studied primarily for the purpose of reaping additional energy savings. Because the early works on the latter issue focused only upon minimization of the total energy cost of the entire site, the resulting arrangements were often infeasible because of the fact that the individual savings may not be reasonably distributed and, therefore, not always acceptable to all participating parties. Although various methods have already been proposed to address this issue based upon game theory, a common weakness of the available approaches is because of the assumption of grass-root designs.[1] In most cases in the real world, the existing plants on an industrial park were built to meet the targeted market demands, which arose during different periods in the past, and each should have already been equipped with a HEN by the time of its completion. In other words, the above benefit allocation problem occurs mainly when a multiplant HEN retrofit project is initiated for the purpose of gaining extra energy savings.

As mentioned above, it is often possible to significantly reduce the total utility cost of two or more standalone chemical plants via interplant heat integration, for example, see Bagajewicz and Rodera,[2] Kralj,[3] and Liew et al.[4,5] Although there are other effective techniques available in the literature, the model-based method is adopted in the present work as the primary HEN synthesis tool because it is in general believed to be more rigorous and thus better equipped for locating the true optimum. Zhang et al.[6] proposed to use a superstructure for building a mixed-integer nonlinear programming (MINLP) model to synthesize the multiplant HEN designs. Chang et al.[7] presented an optimization methodology for interplant heat integration using the intermediate fluid circle(s). On the other hand, it has also been well established that the model-based single-plant HEN synthesis strategies may be classified into two types, that is, the simultaneous and sequential approaches. In the former case, by constructing a superstructure and the corresponding MINLP model, the HEN design with the lowest total annual cost (TAC) can be produced accordingly in a single step.[8] In the latter case, the HEN is generated in three consecutive steps. A linear trans-shipment model is formulated for determining the minimum total utility cost in the first step, while a MILP model with the embedded constraint of

Received: August 4, 2020
Revised: August 26, 2020
Accepted: August 28, 2020
Published: August 28, 2020

ACS Publications

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https://dx.doi.org/10.1021/acs.iecr.0c03829

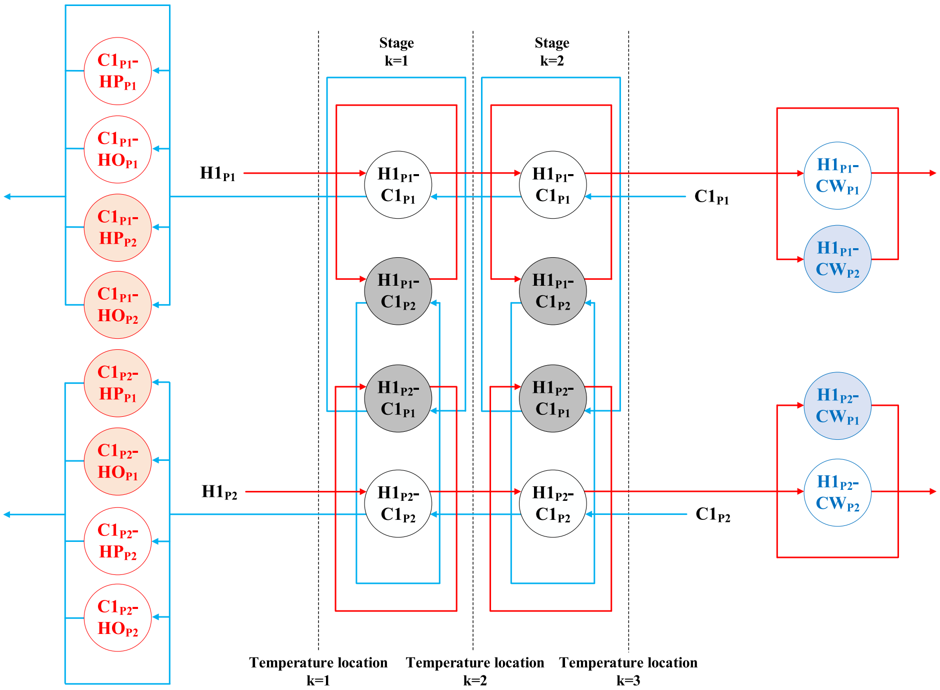

Figure 1. An example of the superstructure for two-plant HEN synthesis.

minimum utility cost is adopted in the second step to identify the minimum number of matches and their heat duties.[9] In the final step of the sequential approach, the NLP model developed by Floudas et al.[10] can be used to synthesize the optimal HEN that minimizes the total capital investment. A modified version of the aforementioned simultaneous HEN synthesis strategy has been formulated in the present study. This practice is because of the need to achieve a better tradeoff between capital and energy costs and, also, because of the easiness in model reformulation so as to facilitate identification of revamp opportunities for interplant heat integration.

As indicated in Cheng et al.[11] many existing interplant heat integration arrangements are often not implementable in practice because of the fact that the profit margin might be unacceptable to one or more participating party. This drawback can be primarily attributed to the conventional HEN design objective, that is, minimization of overall energy cost. Thus, the key to a successful total site heat integration (TSHI) scheme should be to allow every plant obtain reasonable extra benefit while striving for the largest overall saving at the same time. To address this benefit distribution issue, the above authors developed a noncooperative game-based sequential optimization strategy to generate the "fair" interplant integration schemes via direct heat exchanges between the hot and cold process streams across plant boundaries. To further improve the practical feasibility of TSHI projects, Chang et al.[12] modified this sequential optimization approach by replacing the direct interplant heat transfer options with indirect ones. However, it should be noted that all aforementioned strategies are weakened by two obvious drawbacks. First of all, the HEN design produced with the sequential optimization method cannot always reach a true optimum. More importantly, for total-site heat integration, the assumption of noncooperative game may not be valid.

Instead of the noncooperative game, Hiete et al.[13] first treated the above benefit distribution issue as a cooperative game and planned the required interplant heat exchanges based on heuristic judgments. Tan et al.[14] presented a LP model based on cooperative game theory for optimal distribution of cost savings in eco-industrial park. On the other hand, Jin et al.[1] developed a rigorous MINLP-based procedure to handle this allocation problem in the spirit of a cooperative game. Their approach is implemented basically in two stages. The minimum TAC of each and every potential coalition was first determined with a modified version of the conventional MINLP model,[8] while the benefit allocation issue is addressed in the second stage on the basis of the risk-based Shapley values. An effective cost-sharing scheme is constructed in this second stage according to the core solution of a cooperative game[15,16] and the risk-based Shapley values of all players.[17,18] The former ensures coalition stability, while the latter yields a reasonable cost distribution plan.

Although the above methods have been successfully applied to resolve the benefit allocation issue for TSHI, all of them focused on the grass-root designs only. As indicated in the beginning of this section, the benefit allocation problem occurs primarily when a multiplant HEN retrofit project is contemplated for enhancing overall energy efficiency of an existing industrial park. Ciric et al.[19,20] developed a model-based method for determining the optimum retrofit design of the existing HEN configuration in a single plant. Yee and Grossmann[21] proposed a superstructure-based MINLP model to handle the retrofit designs of the single-plant HENs. Soršak and Kravanja[22] described a MINLP model for the retrofit of single-plant HENs that comprise different exchanger types. Ponce-Ortega et al.[23] also presented an optimization model based on the superstructure to produce the redesigned HEN that considers the plant layout. Smith et al.[24] applied the network pinch approach in HEN retrofit design with temperature-dependent thermal properties.

It can be concluded from the above discussions that, although various reliable single-plant HEN retrofit methods have already been proposed, the model-based simultaneous multiplant HEN retrofit design method and the corresponding

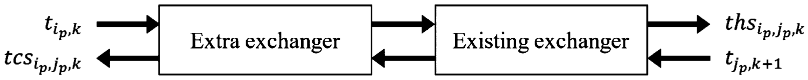

Figure 2. Connections between an existing heat exchanger and a new one to enlarge the heat transfer area.

benefit allocation scheme have never been considered before. Thus, the research objective of the present study is to first formulate and solve a rigorous MINLP model for revamping and merging the existing HENs into a multiplant design via the conventional simultaneous optimization strategy.[8] Because an existing HEN is assumed to be present in each plant, a modified objective function, that is, the extra TAC saving (TACS), is utilized in the model formulation of the multiplant counterpart. The subsequent allocation approach taken in the present study is basically the same as that adopted in Jin et al.[1] In particular, the task of devising the benefit distribution scheme is considered to be analogous to that of engaging in a cooperative game. More specifically, an effective benefit allocation plan is stipulated according to the core of a cooperative game and the Shapley values of all players. Three alternative revamp strategies are devised and compared in a simple example in this paper to demonstrate the feasibility of the proposed design and allocation methods.

2. SUPERSTRUCTURE OF MULTIPLANT HENS

An example of the multiplant counterpart of traditional single-plant superstructure[8] is shown below in Figure 1. This superstructure has been adopted in the present study to address the needs to incorporate design options for placing interplant matches and for consuming external utilities across plant boundaries. For illustration convenience, Figure 1 shows the structure used for two fictitious plants (say P1 and P2). Plant P1 is equipped with two hot utilities (HP_{P1} and HO_{P1}), one cold utility (CW_{P1}), one hot stream ($H1_{P1}$), and one cold stream ($C1_{P1}$), while P2 has two hot utilities (HP_{P2} and HO_{P2}), one cold utility (CW_{P2}), one hot stream ($H1_{P2}$), and one cold stream ($C1_{P2}$). Notice that the interplant and inner-plant matches in this superstructure are represented with gray-colored and uncolored circles, respectively. Because there are a total of two hot streams and two cold streams in the multiplant HEN, the number of stages (denoted as NOK) of this superstructure is set to be 2. Notice also that, in order to introduce revamp flexibility, an extra bypass is placed on each process stream in every stage and also at the end of stream.

The main body of model formulations can be produced on the basis of this superstructure, while additional constraints are imposed to facilitate realization of specific revamp strategies. The former set of equations and inequalities is included in Part A of the Supporting Information for the sake of illustration brevity.

3. REVAMP STRATEGIES

Three revamp strategies have been devised in this work to facilitate interplant heat integration. Basically, each differs from the others mainly in the reclaimed energy and in the capital investments and repiping costs of the resulting multiplant HEN structure. The design guidelines adopted in these three strategies are described in the sequel.

- Strategy I: Only new interplant matches can be introduced into the revamp design. The existing exchanger matches located within each plant must be kept unchanged, while the existing cooler and heater matches may adopt utilities from any plant in the multiplant HEN. Existing coolers and heaters may not be utilized in the revamp design if the corresponding cooling and heating duties are not required. The interplant matches should be housed in new heat exchangers, coolers, or heaters purchased externally. Every inner-plant match should be housed in its original unit and, if a larger heat transfer area is called for, this unit can be connected with an extra new one in series to fulfill the required heat duty (see Figure 2).
- Strategy II: Only new interplant matches are allowed to be introduced into the revamp design. The existing exchanger matches located within each plant must be kept unchanged, while the existing cooler and heater matches may adopt utilities from any plant in the multiplant HEN. Existing coolers and heaters may not be utilized in the revamp design if the corresponding cooling and heating duties are not required. The interplant matches should be housed in new heat exchangers, coolers, or heaters purchased externally. Every inner-plant match can be housed in either its original heat exchanger or another existing one of the same type within the same plant and, if a larger heat transfer area is called for, this unit can be connected with an extra new one in series to fulfill the required heat duty (see Figure 2).
- Strategy III: Both new inner-plant and new interplant matches can be introduced into the revamp design, while some of the existing matches may not be utilized after retrofitting. Every existing heat exchanger should be kept within the plant where it is originally located. Existing coolers and heaters may not be utilized in the revamp design if the corresponding cooling and heating duties are not required. Any match in revamp design can be housed either in a purchased heat exchanger or an existing one and, if a larger heat transfer area is called for in the latter case, this unit can be connected with another new heat exchanger in series to fulfill the required heat duty (see Figure 2).

To facilitate clearer illustration of the above strategies, their unique features are summarized in Table S1, which is placed in Part B of the Supporting Information. Because a greater financial gain can usually be identified by solving a MINLP with more relaxed constraints, it can be expected that Strategy III extracts the most benefit, Strategy I the least, and Strategy II yields a cost saving that lies between the above two. However, it should also be noted that selecting an appropriate strategy actually depends upon additional practical issues, for example, safety concerns, spatial limits, operability, etc.

4. ADDITIONAL CONSTRAINTS TO GENERATE RETROFIT DESIGNS

Other than the constraints described in Part A of the Supporting Information, it is necessary to incorporate additional ones in the programming models for realization of the aforementioned revamp strategies. These constraints are outlined below:

4.1. Extra Constraints Needed for Implementing Strategy I.

In addition to the sets, parameters, and variables defined in Part A of the Supporting Information, more are introduced below to facilitate illustration of the proposed model formulation.

Sets:

$C_{HT}{}^{p}$ set of cold streams that consume hot utilities in the original HEN of plant p.

CL_p set of matches between hot streams that consume cold utilities in the original HEN of plant p and identical cold utilities from any other plant.

CLM_p set of inner-plant matches between hot streams and cold utilities in the original HEN of plant p.

$H_{CL}{}^{p}$ set of hot streams that consume cold utilities in the original HEN of plant p.

HT_p set of matches between cold streams that consume hot utilities in the original HEN of plant p and identical hot utilities from any other plant.

HTM_p set of inner-plant matches between hot utilities and cold streams in the original HEN of plant p.

Y^p set of existing inner-plant matches in the original HEN of plant p.

Parameters:

$A^{EX}_{y_{i_p,j_p}}$ heat transfer area of existing heat exchanger y_{i_p,j_p} ($y_{i_p,j_p}=1,2,...,N_{i_p,j_p}$) used for housing match $(i_p,j_p) \in Y^p$ in the original HEN of plant p.

$A^{EX}_{i_p,n_p}$ heat transfer area of the existing cooler used for housing match $(i_p,n_p) \in CLM^p$ in the original HEN of plant p.

$A^{EX}_{m_p,j_p}$ heat transfer area of the existing heater used for housing match $(m_p,j_p) \in HTM^p$ in the original HEN of plant p.

$LRRC_{j_p,k}$ lower bound for bypass flow fraction of cold stream $j_p \in C^p$ at stage $k \in ST$.

$LRRCU_{j_p}$ lower bound for bypass flow fraction of cold stream $j_p \in C^p$ at the end of stream.

$LRRH_{i_p,k}$ lower bound for bypass flow fraction of hot stream $i_p \in H^p$ at stage $k \in ST$.

$LRRHU_{i_p}$ lower bound for bypass flow fraction of hot stream $i_p \in H^p$ at the end of stream.

LX_{i_p,j_p} lower bound for heat transfer area of the augmented unit for housing match $(i_p,j_p) \in Y^p$ in the multiplant HEN.

$LX_{i_p,n_{q'}}$ lower bound for heat transfer area of the augmented unit for housing cooler match $(i_p,n_{q'}) \in CL_p$ in the multiplant HEN.

LX_{m_q,j_p} lower bound for heat transfer area of the augmented unit for housing heater match $(m_q,j_p) \in HT_p$ in the multiplant HEN.

N_{i_p,j_p} total number of existing heat exchangers used for housing match $(i_p,j_p) \in Y^p$ in the original HEN of plant p.

Λ_{i_p,j_p} a large enough constant which is not smaller than the largest heat transfer area of the augmented unit for housing match $(i_p,j_p) \in Y^p$ in the multiplant HEN.

$\Lambda_{i_p,n_{q'}}$ a large enough constant, which is not smaller than the largest heat transfer area of the augmented unit for housing cooler match $(i_p,n_{q'}) \in CL^p$ in the multiplant HEN.

Λ_{m_q,j_p} a large enough constant, which is not smaller than the largest heat transfer area of the augmented unit for housing heater match $(m_q,j_p) \in HT^p$ in the multiplant HEN.

Variables:

$A_{i_p,j_p,k}$ heat transfer area of match (i_p,j_p) at stage k in the multiplant HEN.

$A_{i_p,n_{q'}}$ heat transfer area of cooler match $(i_p,n_{q'})$ in the multiplant HEN.

A_{m_q,j_p} heat transfer area of heater match (m_q,j_p) in the multiplant HEN.

$e^{y_{i_p,j_p}}_{i_p,j_p,k}$ binary variable used for determining whether or not the existing heat exchanger y_{i_p,j_p} ($y_{i_p,j_p} = 1,2,...,N_{i_p,j_p}$) adopted for housing match $(i_p,j_p) \in Y^p$ in the original HEN of plant p can be used to house the same match at stage k of the multiplant HEN.

$rz_{i_p,k}$ binary variable used for determining whether or not hot stream $i_p \in H^p$ requires a bypass stream at stage k of the multiplant HEN.

$rz_{j_p,k}$ binary variable used for determining whether or not cold stream $j_p \in C^p$ requires a bypass stream at stage k of the multiplant HEN.

$rzcu_{j_p}$ binary variable used for determining whether or not cold stream $j_p \in C^p$ requires a bypass stream for its heater in the multiplant HEN.

$rzhu_{i_p}$ binary variable used for determining whether or not hot stream $i_p \in H^p$ requires a bypass stream for its cooler in the multiplant HEN.

$X^{y_{i_p,j_p}}_{i_p,j_p,k}$ heat transfer area of the augmented unit of the existing heat exchanger y_{i_p,j_p} ($y_{i_p,j_p} = 1,2,...,N_{i_p,j_p}$) adopted for housing match $(i_p,j_p) \in Y^p$ at stage k of the multiplant HEN.

$X_{i_p,n_{q'}}$ heat transfer area of the augmented unit of the existing cooler for housing match $(i_p,n_{q'}) \in CL_p$ in the multiplant HEN.

X_{m_q,j_p} heat transfer area of the augmented unit of the existing heater for housing match $(m_q,j_p) \in HT_p$ in the multiplant HEN.

$\sigma^{y_{i_p,j_p}}_{i_p,j_p,k}$ binary variable used for determining whether or not the existing heat exchanger y_{i_p,j_p} ($y_{i_p,j_p} = 1,2,...,N_{i_p,j_p}$) adopted for housing match $(i_p,j_p) \in Y^p$ in the original HEN of plant p can be used to house the same match at stage k of the multiplant HEN by enlarging its heat transfer area according to Figure 2.

$\sigma_{i_p,n_{q'}}$ binary variable used for determining whether or not the existing cooler for housing match $(i_p,n_{q'}) \in CL_p$ in the original HEN of plant p can be used to house the same match in the multiplant HEN by enlarging its heat transfer area according to Figure 2.

σ_{m_q,j_p} binary variable used for determining whether or not the existing heater for housing match $(m_q,j_p) \in HT_p$ in the original HEN of plant p can be used to house the same match in the multiplant HEN by enlarging its heat transfer area according to Figure 2.

Other than all the constraints mentioned in Part A of the Supporting Information, extra ones should be included for implementation of Strategy I. These model constraints are given in the sequel:

First, to house the existing matches in their original heat exchangers, the following constraints can be imposed

$$\sum_{k \in ST} \xi_{i_p,j_p,k} \le N_{i_p,j_p};\ \ i_p \in H^p;\ \ j_p \in C^p;\ \ (i_p, j_p) \in Y_p \qquad (1)$$

$$\xi^{CU}_{i_p,n_{q'}} \le 1;\ \ i_p \in H^p;\ \ n_{q'} \in \bigcup_{q'=1}^{P} CU^{q'};\ \ (i_p, n_{q'}) \in CL_p \tag{2}$$

$$\xi^{HU}_{m_q,j_p} \le 1;\ \ m_q \in \bigcup_{q=1}^{P} HU^{q};\ \ j_p \in C^p;\ \ (m_q, j_p) \in HT_p \tag{3}$$

where $p = 1,2,...,P$. Note also that the binary variables $\xi_{i_p,j_p,k}$, $\xi^{CU}_{i_p,n_p}$, and $\xi^{HU}_{m_p,j_p}$ should be set to zero if they are not associated with the existing matches, that is,

$$\xi_{i_p,j_p,k} = 0;\ \ i_p \in H^p;\ \ j_p \in C^p;\ \ (i_p, j_p) \notin Y_p \tag{4}$$

$$\xi^{CU}_{i_p,n_p} = 0;\ \ i_p \in H^p;\ \ n_p \in CU^p;\ \ (i_p, n_p) \notin CL_p \tag{5}$$

$$\xi^{HU}_{m_p,j_p} = 0;\ \ m_p \in HU^p;\ \ j_p \in C^p;\ \ (m_p, j_p) \notin HT_p \tag{6}$$

Exactly one existing heat exchanger (which is originally used to house an existing match in the single-plant HEN) can be used to house the same match in a distinct stage in the multiplant HEN, that is

$$\sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}} e^{y_{i_p,j_p}}_{i_p,j_p,k} - \xi_{i_p,j_p,k} = 0;\ \ i_p \in H^p;\ \ j_p \in C^p;\ \ (i_p, j_p) \in Y_p;\ \ k \in ST \tag{7}$$

$$\sum_{k \in ST} e^{y_{i_p,j_p}}_{i_p,j_p,k} = 1;\ \ i_p \in H^p;\ \ j_p \in C^p;\ \ (i_p, j_p) \in Y_p;\ \ y_{i_p,j_p} = 1,2,\ ...,\ N_{i_p,j_p} \tag{8}$$

Also, the heat transfer areas of the units in the multiplant HEN should be constrained as follows:

$$A_{i_p,j_p,k} - A^{EX}_{y_{i_p,j_p}} e^{y_{i_p,j_p}}_{i_p,j_p,k} \ge 0;\ \ i_p \in H^p;\ \ j_p \in C^p;\ \ (i_p, j_p) \in Y_p;\ \ k \in ST;\ \ y_{i_p,j_p} = 1,2,3,\ ...,\ N_{i_p,j_p} \tag{9}$$

$$A_{i_p,n_{q'}} - A^{EX}_{i_p,n_p}\xi^{CU}_{i_p,n_{q'}} \ge 0;\ \ i_p \in H^p;\ \ n_{q'} \in \bigcup_{q'=1}^{P} CU^{q'};\ \ (i_p, n_{q'}) \in CL_p \tag{10}$$

$$A_{m_q,j_p} - A^{EX}_{m_p,j_p}\xi^{HU}_{m_q,j_p} \ge 0;\ \ m_q \in \bigcup_{q=1}^{P} HU^{q};\ \ j_p \in C^p;\ \ (m_q, j_p) \in HT_p \tag{11}$$

Because an inner-plant match in the multiplant HEN is housed in an existing unit, which is used to house the same match in the single-plant HEN and a new heat exchanger may or may not be added according to Figure 2, the heat transfer area of this augmented unit ($X^{y_{i_p,j_p}}_{i_p,j_p,k}$) can be determined as follows

$$X^{y_{i_p,j_p}}_{i_p,j_p,k} = (A_{i_p,j_p,k} - A^{EX}_{y_{i_p,j_p}}) e^{y_{i_p,j_p}}_{i_p,j_p,k};\ \ i_p \in H^p;\ \ j_p \in C^p;\ \ (i_p, j_p) \in Y_p;\ \ k \in ST;\ \ y_{i_p,j_p} = 1,2,3,\ ...,\ N_{i_p,j_p} \tag{12}$$

On the other hand, the inner-plant coolers and heaters in the multiplant HEN are always not smaller than the existing ones due to eqs 10 and 11. Their augmented heat transfer areas ($X_{i_p,n_{q'}}$ and X_{m_q,j_p}) can be expressed as

$$X_{i_p,n_{q'}} = (A_{i_p,n_{q'}} - A^{EX}_{i_p,n_p})\xi^{CU}_{i_p,n_{q'}};\ \ i_p \in H^p;\ \ n_{q'} \in \bigcup_{q'=1}^{P} CU^{q'};\ \ (i_p, n_{q'}) \in CL_p \tag{13}$$

$$X_{m_q,j_p} = (A_{m_q,j_p} - A^{EX}_{m_p,j_p})\xi^{HU}_{m_q,j_p};\ \ m_q \in \bigcup_{q=1}^{P} HU^{q};\ \ j_p \in C^p;\ \ (m_q, j_p) \in HT_p \tag{14}$$

To facilitate the calculation of the capital cost of augmented unit for each existing match in the multiplant HEN, the following logic constraints must be imposed:

$$X^{y_{i_p,j_p}}_{i_p,j_p,k} - \Lambda_{i_p,j_p}\sigma^{y_{i_p,j_p}}_{i_p,j_p,k} \le 0;\ \ i_p \in H^p;\ \ j_p \in C^p;\ \ (i_p, j_p) \in Y^p;\ \ k \in ST;\ \ y_{i_p,j_p} = 1,2,3,\ ...,\ N_{i_p,j_p} \tag{15}$$

$$X_{i_p,n_{q'}} - \Lambda_{i_p,n_{q'}}\sigma_{i_p,n_{q'}} \le 0;\ i_p \in H^p;\ \ n_{q'} \in \bigcup_{q'=1}^{P} CU^{q'};\ \ (i_p, n_{q'}) \in CL_p \tag{16}$$

$$X_{m_q,j_p} - \Lambda_{m_q,j_p}\sigma_{m_q,j_p} \le 0;\ \ m_q \in \bigcup_{q=1}^{P} HU^{q};\ \ j_p \in C^p;\ \ (m_q, j_p) \in HT_p \tag{17}$$

Similarly, to facilitate the calculation of the capital cost of bypasses in the multiplant HEN, the following logic constraints must also be imposed:

$$rrh_{i_p,k} - rz_{i_p,k} \le 0;\ \ i_p \in H^p;\ \ k \in ST \tag{18}$$

$$rrc_{j_p,k} - rz_{j_p,k} \le 0;\ \ j_p \in C^p;\ \ k \in ST \tag{19}$$

$$rrhu_{i_p} - rzhu_{i_p} \le 0;\ \ i_p \in H^p \tag{20}$$

$$rrcu_{j_p} - rzcu_{j_p} \le 0;\ \ j_p \in C^p \tag{21}$$

In addition, to avoid impractically small heat transfer areas of the augmented units, the following constraints should be incorporated:

$$X^{y_{i_p,j_p}}_{i_p,j_p,k} - LX_{i_p,j_p}\sigma^{y_{i_p,j_p}}_{i_p,j_p,k} \ge 0;\ \ i_p \in H^p;\ \ j_p \in C^p;\ \ (i_p, j_p) \in Y^p;\ \ k \in ST;\ \ y_{i_p,j_p} = 1,2,3,\ ...,\ N_{i_p,j_p} \tag{22}$$

$$X_{i_p,n_{q'}} - LX_{i_p,n_{q'}}\sigma_{i_p,n_{q'}} \ge 0;\ i_p \in H^p;\ \ n_{q'} \in \bigcup_{q'=1}^{P} CU^{q'};\ \ (i_p, n_{q'}) \in CL_p \tag{23}$$

$$X_{m_q,j_p} - LX_{m_q,j_p}\sigma_{m_q,j_p} \ge 0;\ \ m_q \in \bigcup_{q=1}^{P} HU^{q};\ \ j_p \in C^p;\ \ (m_q, j_p) \in HT_p \tag{24}$$

The impractically small bypass flow fraction of every bypass should also be prohibited as follows

$$rrh_{i_p,k} - LRRH_{i_p,k} rz_{i_p,k} \geq 0; \;\; i_p \in H^p; \;\; k \in ST \tag{25}$$

$$rrc_{j_p,k} - LRRC_{j_p,k} rz_{j_p,k} \geq 0; \;\; j_p \in C^p; \;\; k \in ST \tag{26}$$

$$rrhu_{i_p} - LRRHU_{i_p} rzhu_{i_p} \geq 0; \;\; i_p \in H^p \tag{27}$$

$$rrcu_{j_p} - LRRCU_{j_p} rzcu_{j_p} \geq 0; \;\; j_p \in C^p \tag{28}$$

4.2. Extra Constraints Needed for Implementing Strategy II. In addition to the sets, parameters, and variables defined in Part A of the Supporting Information and in subsection 4.1, more should be introduced below to facilitate the illustration of the proposed model formulation for implementing Strategy II.

Sets:

$C^p_{HT^{l_p}}$ set of cold streams that consume hot utilities of type l_p in the original HEN of plant p.

$CL^{w_p}_p$ set of existing cooler matches (i_p,n_p) of type w_p in plant p.

CU^{w_p} set of cold utilities of type w_p in plant p.

$H^p_{CL^{w_p}}$ set of hot streams that consume cold utilities of type w_p in the original HEN of plant p.

$HT^{l_p}_p$ set of existing heater matches (m_p,j_p) of type l_p in plant p.

HU^{l_p} set of hot utilities of type l_p in plant p.

Parameters:

L_p total number of heater types in plant p.

N_p total number of existing heat exchangers in the original HEN of plant p, that is, $N_p = \sum_{(i_p,j_p)\in Y^p} N_{i_p,j_p}$.

$N^{w_p}_{CL}$ total number of existing coolers of type w_p in the original HEN of plant p.

$N^{l_p}_{HT}$ total number of existing heaters of type l_p in the original HEN of plant p.

W_p total number of cooler types in plant p.

Variables:

$e^{y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k}$ binary variable used for determining whether or not the existing heat exchanger y_{i_p,j_p} $(y_{i_p,j_p} = 1,2,\ldots,N_{i_p,j_p})$ adopted for housing match $(i_p,j_p) \in Y^p$ in the original HEN of plant p can be used to house another match $(\tilde{i}_p,\tilde{j}_p) \in Y^p$ at stage k of the multiplant HEN.

$e^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_{q'}}$ binary variable used for determining whether or not an existing cooler of type w_p, that is, $(i_p, n_p) \in CL^{w_p}_p$, in the original HEN of plant p can be used to house another cooler match $(\tilde{i}_p,\tilde{n}_{q'})$ in the multiplant HEN.

$e^{m_p,j_p,l_p}_{\tilde{m}_q,\tilde{j}_p}$ binary variable used for determining whether or not an existing heater of type l_p, that is, $(m_p, j_p) \in HT^{l_p}_p$, in the original HEN of plant p can be used to house another heater match $(\tilde{m}_q,\tilde{j}_p)$ in the multiplant HEN.

$X^{y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k}$ heat transfer area of the augmented unit of the existing heat exchanger y_{i_p,j_p} $(y_{i_p,j_p} = 1,2,\ldots,N_{i_p,j_p})$ of match $(i_p,j_p) \in Y^p$ in the original single-plant HEN for housing another match $(\tilde{i}_p,\tilde{j}_p) \in Y^p$ at stage k of the multiplant HEN.

$X^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_{q'}}$ heat transfer area of the augmented unit of the existing cooler of type w_p, that is, $(i_p, n_p) \in CL^{w_p}_p$, in the original HEN of plant p for housing another cooler match $(\tilde{i}_p,\tilde{n}_{q'})$ in the multiplant HEN.

$X^{m_p,j_p,l_p}_{\tilde{m}_q,\tilde{j}_p}$ heat transfer area of the augmented unit of the existing heater of type l_p, that is, $(m_p, j_p) \in HT^{l_p}_p$, in the original HEN of plant p for housing another heater match $(\tilde{m}_q,\tilde{j}_p)$ in the multiplant HEN.

$\sigma^{y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k}$ binary variable used for determining whether or not the existing heat exchanger y_{i_p,j_p} $(y_{i_p,j_p} = 1,2,\ldots,N_{i_p,j_p})$ adopted for housing match $(i_p,j_p) \in Y^p$ in the original HEN of plant p can be used to house another match $(\tilde{i}_p,\tilde{j}_p) \in Y^p$ at stage k of the multiplant HEN by enlarging its heat transfer area according to Figure 2.

$\sigma^{i_p,n_p,w_p}_{\tilde{i}_p,\tilde{n}_{q'}}$ binary variable used for determining whether or not the existing cooler of type w_p, that is, $(i_p, n_p) \in CL^{w_p}_p$, in the original HEN of plant p can be used to house another cooler match $(\tilde{i}_p,\tilde{n}_{q'})$ in the multiplant HEN by enlarging its heat transfer area according to Figure 2.

$\sigma^{m_p,j_p,l_p}_{\tilde{m}_q,\tilde{j}_p}$ binary variable used for determining whether or not the existing heater of type l_p, that is, $(m_p, j_p) \in HT^{l_p}_p$, in the original HEN of plant p can be used to house another heater match $(\tilde{m}_q,\tilde{j}_p)$ in the multiplant HEN by enlarging its heat transfer area according to Figure 2.

Together with all the constraints mentioned in Part A of the Supporting Information, additional ones should be included for implementing Strategy II. The needed constraints are described below:

First of all, for housing the existing exchanger matches of the multiplant HEN with the available heat exchangers, the following inequality should be imposed

$$\sum_{(i_p,j_p)\in Y_p} \sum_{k\in ST} \xi_{i_p,j_p,k} \leq N_p \tag{29}$$

where, $p = 1,2,\ldots,P$. On the other hand, for housing the existing cooler and heater matches of multiplant HEN with the available coolers and heaters of the same types, the following constraints should be used

$$\sum_{q'=1}^{P} \sum_{i_p\in H^p_{CL^{w_p}}} \sum_{n_q'\in CU^{w_q'}} \xi^{CU}_{i_p,n_q'} \leq N^{w_p}_{CL_p}; \;\; w_p = w_q' = 1,2,\ \ldots,\ W_p \tag{30}$$

$$\sum_{q=1}^{P} \sum_{m_q\in HU^{l_q}} \sum_{j_p\in C^p_{HT^{l_p}}} \xi^{HU}_{m_q,j_p} \leq N^{l_p}_{HT_p}; \;\; l_p = l_q = 1,2,\ \ldots,\ L_p \tag{31}$$

In addition, eqs 4−6 are still valid in the present case. Because every existing match can be housed in either its original heat exchanger or another existing one of the same type within the same plant, the corresponding constraints should be written as

$$\sum_{(i_p,j_p)\in Y^p} \sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}} e^{y_{i_p,j_p}}_{\tilde{i}_p,\tilde{j}_p,k} - \xi_{\tilde{i}_p,\tilde{j}_p,k} = 0; \;\; \tilde{i}_p \in H^p; \;\; \tilde{j}_p \in C^p;$$
$$(\tilde{i}_p,\ \tilde{j}_p) \in Y^p; \;\; k \in ST \tag{32}$$

$$\sum_{(i_p,n_p)\in CL_p^{w_p}} e_{\tilde{i}_p,\tilde{n}_q'}^{i_p,n_p,w_p} - \xi_{\tilde{i}_p,\tilde{n}_q'}^{CU} = 0;\ \ \tilde{i}_p \in H_{CL^{w_p}}^{p};\ \ \tilde{n}_q' \in CU^{w_q'};$$

$$w_q' = w_p = 1,2,\ldots,W_p;\ \ q = '1,2,\ldots,P \tag{33}$$

$$\sum_{(m_p,j_p)\in HT_p^{l_p}} e_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} - \xi_{\tilde{m}_q,\tilde{j}_p}^{HU} = 0;\ \ \tilde{m}_q \in HU^{l_q};\ \ \tilde{j}_p \in C_{HT^{l_p}}^{p};$$

$$l_q = l_p = 1,2,\ldots,L_p;\ \ q = 1,2,\ldots,P \tag{34}$$

Furthermore, each existing heat exchanger should be used to house exactly one match in the multiplant HEN, that is

$$\sum_{(\tilde{i}_p,\tilde{j}_p)\in Y^p} \sum_{k\in ST} \mathrm{e}_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} = 1;\ \ i_p \in H^p;\ \ j_p \in C^p;\ \ (i_p,j_p) \in Y^p;$$

$$y_{i_p,j_p} = 1,2,\ldots,N_{i_p,j_p} \tag{35}$$

Also, if the existing units can be adopted in the multiplant HEN, the corresponding heat transfer areas should be constrained as follows

$$A_{\tilde{i}_p,\tilde{j}_p,k} - A_{y_{i_p,j_p}}^{EX} \mathrm{e}_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} \geq 0;\ \ i_p,\tilde{i}_p \in H^p;\ \ j_p,\tilde{j}_p \in C^p;\ \ (i_p,j_p),$$

$$(\tilde{i}_p,\tilde{j}_p) \in Y^p;\ \ k \in ST;\ \ y_{i_p,j_p} = 1,2,\ldots,N_{i_p,j_p} \tag{36}$$

$$A_{\tilde{i}_p,\tilde{n}_{q'}} - A_{i_p,n_p}^{EX} \mathrm{e}_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} \geq 0;\ \ i_p \in H^p;\ \ n_p \in CU^p;\ \ (i_p,n_p) \in CL_p^{w_p};$$

$$\tilde{i}_p \in H_{CL^{w_p}}^{p};\ \ \tilde{n}_{q'} \in CU^{w_{q'}};\ \ w_{q'} = w_p = 1,2,\ldots,W_p;\ \ q' = 1,2,\ldots,P \tag{37}$$

$$A_{\tilde{m}_q,\tilde{j}_p} - A_{m_p,j_p}^{EX} \mathrm{e}_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} \geq 0;\ \ m_p \in HU^p;\ \ j_p \in C^p;\ \ (m_p,j_p)$$

$$\in HT_p^{l_p};\ \ \tilde{m}_q \in HU^{l_q};\ \ \tilde{j}_p \in C_{HT^{l_p}}^{p};\ \ l_q = l_p = 1,2,\ldots,L_p;$$

$$q = 1,2,\ldots,P \tag{38}$$

Because an inner-plant match in the multiplant HEN may be housed in any existing unit and a new heat exchanger may or may not be added according to Figure 2, the heat transfer area of the augmented units ($X_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}}$, $X_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p}$, and $X_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p}$) can be determined using the following equations

$$X_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} = (A_{\tilde{i}_p,\tilde{j}_p,k} - A_{y_{i_p,j_p}}^{EX})\mathrm{e}_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}};\ \ i_p,\tilde{i}_p \in H^p;$$

$$j_p,\tilde{j}_p \in C^p;\ \ (i_p,j_p),(\tilde{i}_p,\tilde{j}_p) \in Y^p;\ \ k \in ST;$$

$$y_{i_p,j_p} = 1,2,\ldots,N_{i_p,j_p} \tag{39}$$

$$X_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} = (A_{\tilde{i}_p,\tilde{n}_{q'}} - A_{i_p,n_p}^{EX})\mathrm{e}_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p};\ \ i_p \in H^p;\ \ n_p \in CU^p;$$

$$(i_p,n_p) \in CL_p^{w_p};\ \ \tilde{i}_p \in H_{CL^{w_p}}^{p};\ \ \tilde{n}_{q'} \in CU^{w_{q'}};$$

$$w_{q'} = w_p = 1,2,\ldots,W_p;\ \ q' = 1,2,\ldots,P \tag{40}$$

$$X_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} = (A_{\tilde{m}_q,\tilde{j}_p} - A_{m_p,j_p}^{EX})\mathrm{e}_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p};\ m_p \in HU^p;\ \ j_p \in C^p;$$

$$(m_p,j_p) \in HT_p^{l_p};\ \ \tilde{m}_q \in HU^{l_q};\ \ \tilde{j}_p \in C_{HT^{l_p}}^{p};$$

$$l_q = l_p = 1,2,\ldots,L_p;\ \ q = 1,2,\ldots,P \tag{41}$$

To facilitate calculation of the capital cost of augmented unit for each existing match in the multiplant HEN, the following logic constraints must be imposed

$$X_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} - \Lambda_{\tilde{i}_p,\tilde{j}_p}\sigma_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} \leq 0;\ i_p,\tilde{i}_p \in H^p;\ \ j_p,\tilde{j}_p \in C^p;$$

$$(i_p,j_p),(\tilde{i}_p,\tilde{j}_p) \in Y^p;\ \ k \in ST;\ \ y_{i_p,j_p} = 1,2,\ldots,N_{i_p,j_p} \tag{42}$$

$$X_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} - \Lambda_{\tilde{i}_p,\tilde{n}_{q'}}\sigma_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} \leq 0;\ i_p \in H^p;\ \ n_p \in CU^p;$$

$$(i_p,n_p) \in CL_p^{w_p};\ \ \tilde{i}_p \in H_{CL^{w_p}}^{p};\ \ \tilde{n}_{q'} \in CU^{w_{q'}};$$

$$w_{q'} = w_p = 1,2,\ldots,W_p;\ \ q' = 1,2,\ldots,P \tag{43}$$

$$X_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} - \Lambda_{\tilde{m}_q,\tilde{j}_p}\sigma_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} \leq 0;\ \ m_p \in HU^p;\ \ j_p \in C^p;$$

$$(m_p,j_p) \in HT_p^{l_p};\ \ \tilde{m}_q \in HU^{l_q};\ \ \tilde{j}_p \in C_{HT^{l_p}}^{p};$$

$$l_q = l_p = 1,2,\ldots,L_p;\ \ q = 1,2,\ldots,P \tag{44}$$

Furthermore, to avoid using impractically small heat transfer areas of the augmented units, the following constraints should be incorporated

$$X_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} - LX_{\tilde{i}_p,\tilde{j}_p}\sigma_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} \geq 0;\ i_p,\tilde{i}_p \in H^p;\ \ j_p,\tilde{j}_p \in C^p;$$

$$(i_p,j_p),(\tilde{i}_p,\tilde{j}_p) \in Y^p;\ \ k \in ST;\ \ y_{i_p,j_p} = 1,2,\ldots,N_{i_p,j_p} \tag{45}$$

$$X_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} - LX_{\tilde{i}_p,\tilde{n}_{q'}}\sigma_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} \geq 0;\ i_p \in H^p;\ \ n_p \in CU^p;$$

$$(i_p,n_p) \in CL_p^{w_p};\ \ \tilde{i}_p \in H_{CL^{w_p}}^{p};\ \ \tilde{n}_{q'} \in CU^{w_{q'}};$$

$$w_{q'} = w_p = 1,2,\ldots,W_p;\ \ q' = 1,2,\ldots,P \tag{46}$$

$$X_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} - LX_{\tilde{m}_q,\tilde{j}_p}\sigma_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} \geq 0;\ m_p \in HU^p;\ \ j_p \in C^p;$$

$$(m_p,j_p) \in HT_p^{l_p};\ \ \tilde{m}_q \in HU^{l_q};\ \ \tilde{j}_p \in C_{HT^{l_p}}^{p};$$

$$l_q = l_p = 1,2,\ldots,L_p;\ \ q = 1,2,\ldots,P \tag{47}$$

Finally, notice that eqs 18–21 and 25–28 should also be included in the present application.

4.3. Extra Constraints Needed for Implementing Strategy III.

In addition to the sets, parameters, and variables defined in Part A of the Supporting Information and in subsections 4.1 and 4.2, extra definitions are introduced below to facilitate model formulation for implementing Strategy III.

Sets:

Z_p set of all possible inner-plant matches between hot stream $\tilde{i}_p \in H^p$ and cold stream $\tilde{j}_p \in C^p$ in plant p.

$ZCL_p^{w_p}$ set of all possible cooler matches between hot stream $\tilde{i}_p \in H^p$ and cold utility $\tilde{n}_p \in CU^p$, which can be housed in coolers of type w_p.

$ZHT_p^{l_p}$ set of all possible heater matches between hot utility $\tilde{m}_p \in H^p$ and cold stream $\tilde{j}_p \in C^p$, which can be housed in heaters of type l_p.

Variables:

$\mathrm{e}_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}}$ binary variable used for determining whether or not the existing heat exchanger y_{i_p,j_p} ($y_{i_p,j_p} = 1,2,\ldots,N_{i_p,j_p}$) adopted for housing match $(i_p,j_p) \in Y^p$ in the original HEN of plant p can be used to house another match $(\tilde{i}_p,\tilde{j}_p) \in Z^p$ at stage k of the multiplant HEN.

$\mathrm{e}_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p}$ binary variable used for determining whether or not an existing cooler of type w_p, that is, $(i_p, n_p) \in CL_p^{w_p}$, in the original HEN of plant p can be used to house another cooler match $(\tilde{i}_p,\tilde{n}_{q'})$ in the multiplant HEN.

$\mathrm{e}_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p}$ binary variable used for determining whether or not an existing heater of type l_p, that is, $(m_p, j_p) \in HT_p^{l_p}$, in the original HEN of plant p can be used to house another heater match $(\tilde{m}_q,\tilde{j}_p)$ in the multiplant HEN.

$u_{\tilde{i}_p,\tilde{j}_p,k}$ binary variable used for determining whether or not a new heat exchanger should be purchased to house match $(\tilde{i}_p,\tilde{j}_p) \in Z^p$ at stage k of the multiplant HEN.

$u_{\tilde{i}_p,\tilde{n}_p}^{CU,w_p}$ binary variable used for determining whether or not a new cooler of type w_p should be purchased to house cooler match $(\tilde{i}_p, \tilde{n}_p) \in ZCL_p^{w_p}$ in the multiplant HEN.

$u_{\tilde{m}_p,\tilde{j}_p}^{HU,l_p}$ binary variable used for determining whether or not a new heater of type l_p should be purchased to house heater match $(\tilde{m}_p, \tilde{j}_p) \in ZHT_p^{l_p}$ in the multiplant HEN.

$v_{\tilde{i}_p,\tilde{n}_{q'}}^{CU,w_p}$ binary variable used for determining whether or not a new cooler of type w_p should be purchased to house interplant cooler match $(\tilde{i}_p,\tilde{n}_{q'})$ in the multiplant HEN.

$v_{\tilde{m}_q,\tilde{j}_p}^{HU,l_p}$ binary variable used for determining whether or not a new heater of type l_p should be purchased to house interplant heater match $(\tilde{m}_q,\tilde{j}_p)$ in the multiplant HEN.

$X_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}}$ heat transfer area of the augmented unit for the existing heat exchanger $y_{ip,jp}$ $(y_{ip,jp}=1,2,\ldots,N_{ip,jp})$ of match $(i_p,j_p) \in Y^p$ in the original single-plant HEN used for housing another match $(\tilde{i}_p,\tilde{j}_p) \in Z^p$ at stage k in the multiplant HEN.

$X_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p}$ heat transfer area of the augmented unit of the existing cooler of type w_p, that is, $(i_p, n_p) \in CL_p^{w_p}$, in the original HEN of plant p for housing another cooler match $(\tilde{i}_p,\tilde{n}_{q'})$ in the multiplant HEN.

$X_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p}$ heat transfer area of the augmented unit of the existing heater of type l_p, that is, $(m_p, j_p) \in HT_p^{l_p}$, in the original HEN of plant p for housing another heater match $(\tilde{m}_q,\tilde{j}_p)$ in the multiplant HEN.

$\sigma_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}}$ binary variable used for determining whether or not the existing heat exchanger y_{i_p,j_p} $(y_{i_p,j_p} = 1,2,\ldots,N_{i_p,j_p})$ originally adopted for housing match $(i_p,j_p) \in Y^p$ in the single-plant HEN of plant p can be used to house another match $(\tilde{i}_p,\tilde{j}_p) \in Z^p$ at stage k of the multiplant HEN by enlarging its heat transfer area according to Figure 2.

$\sigma_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p}$ binary variable used for determining whether or not the existing cooler of type w_p, that is, $(i_p, n_p) \in CL_p^{w_p}$, in the original HEN of plant p can be used to house another cooler match $(\tilde{i}_p,\tilde{n}_{q'})$ in the multiplant HEN by enlarging its heat transfer area according to Figure 2.

$\sigma_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p}$ binary variable used for determining whether or not the existing heater of type l_p, that is, $(m_p, j_p) \in HT_p^{l_p}$, in the original HEN of plant p can be used to house another heater match $(\tilde{m}_q,\tilde{j}_p)$ in the multiplant HEN by enlarging its heat transfer area according to Figure 2.

Other than all the constraints mentioned in Part A of the Supporting Information, additional ones should be included for implementation of Strategy III. Such constraints are given below:

For housing the exchanger matches of the multiplant HEN with either available or new heat exchangers, the following inequality should be imposed

$$\sum_{(i_p,j_p)\in Y^p} \sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}} \mathrm{e}_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} + u_{\tilde{i}_p,\tilde{j}_p,k} - \xi_{\tilde{i}_p,\tilde{j}_p,k} = 0;$$

$$\tilde{i}_p \in H^p;\ \tilde{j}_p \in C^p;\ (\tilde{i}_p, \tilde{j}_p) \in Z^p;\ k \in ST \tag{48}$$

On the other hand, for housing the cooler and heater matches in multiplant HEN with either available or new coolers and heaters of the same types, the following constraints should be used

$$\sum_{(i_p,n_p)\in CL_p^{w_p}} \mathrm{e}_{\tilde{i}_p,\tilde{n}_p}^{i_p,n_p,w_p} + u_{\tilde{i}_p,\tilde{n}_p}^{CU,w_p} - \xi_{\tilde{i}_p,\tilde{n}_p}^{CU} = 0;\ \tilde{i}_p \in H^p;$$

$$\tilde{n}_p \in CU^p;\ (\tilde{i}_p, \tilde{n}_p) \in ZCL_p^{w_p};\ w_p = 1,2, \ldots, W_p \tag{49}$$

$$\sum_{(i_p,n_p)\in CL_p^{w_p}} \mathrm{e}_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} + v_{\tilde{i}_p,\tilde{n}_{q'}}^{CU,w_p} - \xi_{\tilde{i}_p,\tilde{n}_{q'}}^{CU} = 0;\ \tilde{i}_p \in H^p;$$

$$\tilde{n}_{q'} \in CU^{w_{q'}};\ w_p = w_{q'} = 1,2, \ldots,\ W_p;\ q' \neq p \tag{50}$$

$$\sum_{(m_p,j_p)\in HT_p^{l_p}} \mathrm{e}_{\tilde{m}_p,\tilde{j}_p}^{m_p,j_p,l_p} + u_{\tilde{m}_p,\tilde{j}_p}^{HU,l_p} - \xi_{\tilde{m}_p,\tilde{j}_p}^{HU} = 0;\ \tilde{m}_p \in HU^p;$$

$$\tilde{j}_p \in C^p;\ (\tilde{m}_p, \tilde{j}_p) \in ZHT_p^{l_p};\ l_p = 1,2, \ldots, L_p \tag{51}$$

$$\sum_{(m_p,j_p)\in HT_p^{l_p}} \mathrm{e}_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} + v_{\tilde{m}_q,\tilde{j}_p}^{HU} - \xi_{\tilde{m}_q,\tilde{j}_p}^{HU} = 0;$$

$$\tilde{m}_q \in HU^{l_q};\ \tilde{j}_p \in C^p;\ l_p = l_q = 1,2, \ldots, L_p;\ q \neq p \tag{52}$$

In the multiplant HEN, the total numbers of heat exchangers, coolers, and heaters that are housed in existing units should be bounded, that is

$$\sum_{(i_p,j_p)\in Y^p} \sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}} \sum_{(\tilde{i}_p,\tilde{j}_p)\in Z^p} \sum_{k\in ST} \mathrm{e}_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} \leq N_p \tag{53}$$

$$\sum_{q'=1}^{P} \sum_{(i_p,n_p)\in CL_p^{w_p}} \sum_{\tilde{i}_p\in H^p} \sum_{\tilde{n}_{q'}\in CU^{w_{q'}}} \mathrm{e}_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} \leq N_{CL}^{w_p};$$

$$w_p = w_{q'} = 1,2, \ldots, W_p \tag{54}$$

$$\sum_{q=1}^{P} \sum_{(m_p,j_p)\in HT_p^{l_p}} \sum_{\tilde{m}_q\in HU^{l_q}} \sum_{\tilde{j}_p\in C^p} \mathrm{e}_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} \leq N_{HT}^{l_p};$$

$$l_p = l_q = 1,2, \ldots, L_p \tag{55}$$

Every existing heat exchanger should of course be used to house exactly one exchanger match in the multiplant HEN, that is

https://dx.doi.org/10.1021/acs.iecr.0c03829
Ind. Eng. Chem. Res. 2020, 59, 18088−18105

$$\sum_{(\tilde{i}_p,\tilde{j}_p)\in Z^p}\ \sum_{k\in ST} \mathrm{e}_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} = 1;\ \ (i_p,\ j_p) \in Y^p;\ \ y_{i_p,j_p} = 1,2,\ ...,\ N_{i_p,j_p} \tag{56}$$

If the existing units can be adopted in the multiplant HEN, the corresponding heat transfer areas should be constrained as follows

$$\begin{aligned} &A_{\tilde{i}_p,\tilde{j}_p,k} - A_{y_{i_p,j_p}}^{EX}\,\mathrm{e}_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} \ge 0;\ i_p,\ \tilde{i}_p \in H^p;\ \ j_p,\ \tilde{j}_p \in C^p;\\ &\quad (i_p,\ j_p) \in Y^p;\ \ (\tilde{i}_p,\ \tilde{j}_p) \in Z^p;\ \ k \in ST;\ \ y_{i_p,j_p} = 1,2,\ ...,\ N_{i_p,j_p} \end{aligned} \tag{57}$$

$$\begin{aligned} &A_{\tilde{i}_p,\tilde{n}_{q'}} - A_{i_p,n_p}^{EX}\mathrm{e}_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} \ge 0;\ \ i_p,\ \tilde{i}_p \in H^p;\ \ n_p \in CU^p;\\ &\quad (i_p,\ n_p) \in CL_p^{w_p};\ \ \tilde{n}_{q'} \in CU^{w_{q'}};\ \ w_{q'} = w_p = 1,2,\ ...,\ W_p;\\ &\quad q' = 1,2,\ ...,\ P \end{aligned} \tag{58}$$

$$\begin{aligned} &A_{\tilde{m}_q,\tilde{j}_p} - A_{m_p,j_p}^{EX}\mathrm{e}_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} \ge 0;\ \ m_p \in HU^p;\ \ j_p,\ \tilde{j}_p \in C^p;\\ &\quad (m_p,\ j_p) \in HT_p^{l_p};\ \ \tilde{m}_q \in HU^{l_q};\ \ \tilde{j}_p \in C^p;\ \ l_q = l_p = 1,2,\ ...,\ L_p;\\ &\quad q = 1,2,\ ...,\ P \end{aligned} \tag{59}$$

The heat transfer areas of the augmented units can be expressed as

$$\begin{aligned} &X_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} = (A_{\tilde{i}_p,\tilde{j}_p,k} - A_{y_{i_p,j_p}}^{EX})\mathrm{e}_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}};\ i_p,\ \tilde{i}_p \in H^p;\\ &\quad j_p,\ \tilde{j}_p \in C^p;\ \ (i_p,\ j_p) \in Y^p;\ \ (\tilde{i}_p,\ \tilde{j}_p) \in Z^p;\ \ k \in ST;\\ &\quad y_{i_p,j_p} = 1,2,\ ...,\ N_{i_p,j_p} \end{aligned} \tag{60}$$

$$\begin{aligned} &X_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} = (A_{\tilde{i}_p,\tilde{n}_{q'}} - A_{i_p,n_p}^{EX})\mathrm{e}_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p};\ i_p,\ \tilde{i}_p \in H^p;\ \ n_p \in CU^p;\\ &\quad (i_p,\ n_p) \in CL_p^{w_p};\ \ \tilde{n}_{q'} \in CU^{w_{q'}};\ \ w_{q'} = w_p = 1,2,\ ...,\ W_p;\\ &\quad q' = 1,2,\ ...,\ P \end{aligned} \tag{61}$$

$$\begin{aligned} &X_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} = (A_{\tilde{m}_q,\tilde{j}_p} - A_{m_p,j_p}^{EX})\mathrm{e}_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p};\ m_p \in HU^p;\\ &\quad j_p,\ \tilde{j}_p \in C^p;\ \ (m_p,\ j_p) \in HT_p^{l_p};\ \ \tilde{m}_q \in HU^{l_q};\\ &\quad l_q = l_p = 1,2,\ ...,\ L_p;\ \ q = 1,2,\ ...,\ P \end{aligned} \tag{62}$$

The presence (or absence) of each augment unit can be determined with the binary variables in the following logic constraints

$$\begin{aligned} &X_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} - \Lambda_{\tilde{i}_p,\tilde{j}_p}\sigma_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} \le 0;\ \ i_p,\ \tilde{i}_p \in H^p;\ \ j_p,\ \tilde{j}_p \in C^p;\\ &\quad (i_p,\ j_p) \in Y^p;\ \ (\tilde{i}_p,\ \tilde{j}_p) \in Z_p;\ \ k \in ST;\ \ y_{i_p,j_p} = 1,2,\ ...,\ N_{i_p,j_p} \end{aligned} \tag{63}$$

$$\begin{aligned} &X_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} - \Lambda_{\tilde{i}_p,\tilde{n}_{q'}}\sigma_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} \le 0;\ i_p,\ \tilde{i}_p \in H^p;\ \ n_p \in CU^p;\\ &\quad (i_p,\ n_p) \in CL_p^{w_p};\ \ \tilde{n}_{q'} \in CU^{w_{q'}};\ \ w_{q'} = w_p = 1,2,\ ...,\ W_p;\\ &\quad q' = 1,2,\ ...,\ P \end{aligned} \tag{64}$$

$$\begin{aligned} &X_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} - \Lambda_{\tilde{m}_q,\tilde{j}_p}\sigma_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} \le 0;\ m_p \in HU^p;\ \ j_p,\ \tilde{j}_p \in C^p;\\ &\quad (m_p,\ j_p) \in HT_p^{l_p};\ \ \tilde{m}_q \in HU^{l_q};\ \ \tilde{j}_p \in C^p;\ \ l_q = l_p = 1,2,\ ...,\ L_p;\\ &\quad q = 1,2,\ ...,\ P \end{aligned} \tag{65}$$

To avoid using impractically small heat transfer areas of the augmented units, the following constraints should be incorporated

$$\begin{aligned} &X_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} - LX_{\tilde{i}_p,\tilde{j}_p}\sigma_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} \ge 0;\ \ i_p,\ \tilde{i}_p \in H^p;\ \ j_p,\ \tilde{j}_p \in C^p;\\ &\quad (i_p,\ j_p) \in Y^p;\ \ (\tilde{i}_p,\ \tilde{j}_p) \in Z^p;\ \ k \in ST;\ \ y_{i_p,j_p} = 1,2,\ ...,\ N_{i_p,j_p} \end{aligned} \tag{66}$$

$$\begin{aligned} &X_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} - LX_{\tilde{i}_p,\tilde{n}_{q'}}\sigma_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} \ge 0;\ i_p,\ \tilde{i}_p \in H^p;\ \ n_p \in CU^p;\\ &\quad (i_p,\ n_p) \in CL_p^{w_p};\ \ \tilde{n}_{q'} \in CU^{w_{q'}};\ \ w_{q'} = w_p = 1,2,\ ...,\ W_p;\\ &\quad q' = 1,2,\ ...,\ P \end{aligned} \tag{67}$$

$$\begin{aligned} &X_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} - LX_{\tilde{m}_q,\tilde{j}_p}\sigma_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} \ge 0;\ m_p \in HU^p;\ \ j_p,\ \tilde{j}_p \in C^p;\\ &\quad (m_p,\ j_p) \in HT_p^{l_p};\ \ \tilde{m}_q \in HU^{l_q};\ \ \tilde{j}_p \in C^p;\ \ l_q = l_p = 1,2,\ ...,\ L_p;\\ &\quad q = 1,2,\ ...,\ P \end{aligned} \tag{68}$$

Finally, the presence (or absence) of bypasses can also be determined with the binary variables in eqs 18−21, while the impractically small bypass flow fractions should be prohibited in eqs 25−28.

5. OBJECTIVE FUNCTIONS

In this study, the overall saving achieved by retrofitting and building the multiplant HEN is used as the objective function to be maximized. This function (TACS) can be expressed as follows

$$\begin{aligned} \mathrm{TACS} = \mathrm{TUC} - \mathrm{TUC}' - \mathrm{af}(&\mathrm{ATCC} + \mathrm{NTCC}_1\\ &+ \mathrm{NTCC}_2 + \mathrm{NTCC}_3 + \mathrm{NTCC}_4 + \mathrm{TUPC}) \end{aligned} \tag{69}$$

where TUC denotes the sum of utility costs of all single-plant HENs, which should be regarded as a given constant in the corresponding MINLP models; TUC′ denotes the total utility cost of the multiplant HEN after retrofit; af is the annualization factor, which is another given constant; ATCC is the total capital cost of all augmented units; NTCC_1 denotes the total capital cost of all new units purchased for interplant matches; NTCC_2 denotes the total capital cost of all bypasses; NTCC_3 denotes the unit reassignment cost; NTCC_4 denotes the total capital cost of all new units purchased for inner-plant matches; and TUPC is the total capital cost of pipes if the existing coolers and heaters adopt utilities from other plant in the multiplant HEN. Other than the aforementioned two constants (i.e., TUC and af), the detailed expressions of the remaining cost models are listed in the subsequent subsections and, for the sake of brevity, all embedded model parameters (or cost coefficients) are first defined below:

Parameters:

BY_{i_p} cost of a single bypass on hot stream i_p.

BY_{j_p} cost of a single bypass on cold stream j_p.

$CA_{i_q j_{q'}}$ variable cost coefficient in the cost model of heat exchanger between hot stream i_q and cold stream $j_{q'}$ ($q,q' = 1,2,...,P$).

$CA_{i_p n_{q'}}$ variable cost coefficient in the cost model of cooler between hot stream i_p and cold utility $n_{q'}$ ($q' = 1,2,...,P$).

$CA_{m_q j_p}$ variable cost coefficient in the cost model of heater between hot utility m_q ($q = 1,2,...,P$) and cold stream j_p.

$CF_{i_q,j_{q'}}$ fixed cost in the cost model of heat exchanger between hot stream i_q and cold stream $j_{q'}$ $(q,q' = 1,2,...,P)$.

$CF_{i_p,n_{q'}}$ fixed cost in the cost model of cooler between hot stream i_p and cold utility $n_{q'}$ $(q' = 1,2,...,P)$.

CF_{m_q,j_p} fixed cost in the cost model of heater between hot utility m_q $(q = 1,2,...,P)$ and cold stream j_p.

$CM_{\tilde{i}_p,\tilde{j}_p}^{i_p,j_p}$ reassignment cost for existing heat exchanger, which houses match (i_p,j_p) in the original HEN of plant p and houses different match $(\tilde{i}_p,\tilde{j}_p)$ in the multiplant HEN.

$CM_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p}$ reassignment cost for existing cooler of type w_p, which houses match (i_p,n_p) in the original HEN of plant p and is utilized by different hot process stream $\tilde{i}_p$ in the multiplant HEN.

$CM_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p}$ reassignment cost for the existing heater of type l_p, which houses match (m_p,j_p) in the original HEN of plant p and is utilized by different cold process stream $\tilde{j}_p$ in the multiplant HEN.

$CP_{i_p,n_{q'}}$ fixed cost of pipe for the existing cooler, which can house interplant match $(i_p,n_{q'})$ in the multiplant HEN by adopting utilities from other plant.

CP_{m_q,j_p} fixed cost of pipe for existing heater, which can house interplant match (m_q,j_p) in the multiplant HEN by adopting utilities from other plant.

$CQ_{i_p,n_{q'}}$ unit cost of cold utility $n_{q'}$ $(q' = 1,2,...,P)$ for cooling hot stream i_p.

CQ_{m_q,j_p} unit cost of hot utility m_q $(q = 1,2,...,P)$ for heating cold stream j_p.

β exponent of heat transfer areas in variable cost terms in the cost models of heat exchanger, cooler, and heater.

5.1. Cost Models Utilized for Implementation of Strategy I. For Strategy I, the cost items embedded in eq 69 are presented in detail as follows

$$\mathrm{TUC}' = \sum_{p=1}^{P}\sum_{q'=1}^{P}\sum_{i_p\in H^p}\sum_{n_{q'}\in CU^{q'}} CQ_{i_p,n_{q'}}q_{i_p,n_{q'}} + \sum_{q=1}^{P}\sum_{p=1}^{P}\sum_{m_q\in HU^q}\sum_{j_p\in C^p} CQ_{m_q,j_p}q_{m_q,j_p} \tag{70}$$

$$\begin{aligned}\mathrm{ATCC} = {} & \sum_{p=1}^{P}\sum_{(i_p,j_p)\in Y^p}\sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}}\sum_{k\in ST}[CF_{i_p,j_p}\sigma_{i_p,j_{p,k}}^{y_{i_p,j_p}} + CA_{i_p,j_p}(X_{i_p,j_{p,k}}^{y_{i_p,j_p}})^{\beta}] \\ & + \sum_{p=1}^{P}\sum_{q'=1}^{P}\sum_{i_p\in H_{CL}^{p}}\sum_{n_{q'}\in CU^{q'}}\sum_{(i_p,n_{q'})\in CL_p}[CF_{i_p,n_{q'}}\sigma_{i_p,n_{q'}} + CA_{i_p,n_{q'}} \\ & (X_{i_p,n_{q'}})^{\beta}] + \sum_{q=1}^{P}\sum_{p=1}^{P}\sum_{m_q\in HU^q}\sum_{j_p\in C_{HT}^{p}}\sum_{(m_q,j_p)\in HT_p}[CF_{m_q,j_p}\sigma_{m_q,j_p} \\ & + CA_{m_q,j_p}(X_{m_q,j_p})^{\beta}]\end{aligned} \tag{71}$$

$$\begin{aligned}\mathrm{NTCC}_1 = {} & \sum_{p=1}^{P}\sum_{q'=1q'\neq p}^{P}\sum_{i_p\in H^p}\sum_{j_{q'}\in C^{q'}}\sum_{k\in ST}[CF_{i_p,j_{q'}}\xi_{i_p,j_{q'},k} \\ & + CA_{i_p,j_{q'}}(A_{i_p,j_{q'}})^{\beta}] + \sum_{p=1}^{P}\sum_{q'=1q'\neq p}^{P}\sum_{i_p\notin H_{CL}^{p}}\sum_{n_{q'}\in CU^{q'}}[CF_{i_p,n_{q'}}\xi_{i_p,n_{q'}} \\ & + CA_{i_p,n_{q'}}(A_{i_p,n_{q'}})^{\beta}] + \sum_{q=1q\neq p}^{P}\sum_{p=1}^{P}\sum_{m_q\in HU^q}\sum_{j_p\notin C_{HT}^{p}}[CF_{m_q,j_p}\xi_{m_q,j_p} \\ & + CA_{m_q,j_p}(A_{m_q,j_p})^{\beta}]\end{aligned} \tag{72}$$

$$\begin{aligned}\mathrm{NTCC}_2 = {} & \sum_{p=1}^{P}\sum_{i_p\in H^p}\left[BY_{i_p}\left(rzhu_{i_p} + \sum_{k\in ST} rz_{i_p,k}\right)\right] \\ & + \sum_{p=1}^{P}\sum_{j_p\in C^p}\left[BY_{j_p}\left(rzcu_{j_p} + \sum_{k\in ST} rz_{j_p,k}\right)\right]\end{aligned} \tag{73}$$

$$\mathrm{NTCC}_3 = 0 \tag{74}$$

$$\mathrm{NTCC}_4 = 0 \tag{75}$$

$$\begin{aligned}\mathrm{TUPC} = {} & \sum_{p=1}^{P}\sum_{q'=1q'\neq p}^{P}\sum_{i_p\in H_{CL}^{p}}\sum_{n_{q'}\in CU^{q'}}(CP_{i_p,n_{q'}}\xi_{i_p,n_{q'}}) \\ & + \sum_{q=1q\neq p}^{P}\sum_{p=1}^{P}\sum_{m_q\in HU^q}\sum_{j_p\in C_{HT}^{p}}(CP_{m_q,j_p}\xi_{m_q,j_p})\end{aligned} \tag{76}$$

5.2. Cost Models Utilized for Implementation of Strategy II. The above cost models are all applicable in the present scenario except ATCC. In particular, eqs 71 and 74 should be replaced by the following formula

$$\begin{aligned}\mathrm{ATCC} = {} & \sum_{p=1}^{P}\sum_{(i_p,j_p)\in Y^p}\sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}}\sum_{(\tilde{i}_p,\tilde{j}_p)\in Y^p}\sum_{k\in ST}[CF_{\tilde{i}_p,\tilde{j}_p}\sigma_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} \\ & + CA_{\tilde{i}_p,\tilde{j}_p}(X_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}})^{\beta}] \\ & + \sum_{p=1}^{P}\sum_{q'=1}^{P}\sum_{w_p=w_{q'}=1}^{W_p}\sum_{(i_p,n_p)\in CL_p^{w_p}}\sum_{\tilde{i}_p\in H_{CL^{w_p}}^{p}}\sum_{\tilde{n}_{q'}\in CU^{w_{q'}}}[CF_{\tilde{i}_p,\tilde{n}_{q'}}\sigma_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} \\ & + CA_{\tilde{i}_p,\tilde{n}_{q'}}(X_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p})^{\beta}] \\ & + \sum_{q=1}^{P}\sum_{p=1}^{P}\sum_{l_p=l_q=1}^{L_p}\sum_{(m_p,j_p)\in HT_p^{l_p}}\sum_{\tilde{m}_q\in HU^{l_q}}\sum_{\tilde{j}_p\in C_{HT^{l_p}}^{p}}[CF_{\tilde{m}_q,\tilde{j}_p}\sigma_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} \\ & + CA_{\tilde{m}_q,\tilde{j}_p}(X_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p})^{\beta}]\end{aligned} \tag{77}$$

$$\begin{aligned}\mathrm{NTCC}_3 = {} & \sum_{p=1}^{P}\sum_{(i_p,j_p)\in Y^p}\sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}}\sum_{(\tilde{i}_p,\tilde{j}_p)\in Y^p\tilde{i}_p\neq i_p\vee\tilde{j}_p\neq j_p}\sum_{k\in ST} CM_{\tilde{i}_p,\tilde{j}_p}^{i_p,j_p}e_{\tilde{i}_p,\tilde{j}_p,k}^{i_p,j_p,y_{i_p,j_p}} \\ & + \sum_{p=1}^{P}\sum_{q'=1}^{P}\sum_{w_p=w_{q'}=1}^{W_p}\sum_{(i_p,n_p)\in CL_p^{w_p}}\sum_{\tilde{i}_p\in H_{CL^{w_p}}^{p}\tilde{i}_p\neq i_p}\sum_{\tilde{n}_{q'}\in CU^{w_{q'}}} CM_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p}e_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} \\ & + \sum_{q=1}^{P}\sum_{p=1}^{P}\sum_{l_p=l_q=1}^{L_p}\sum_{(m_p,j_p)\in HT_p^{l_p}}\sum_{\tilde{m}_q\in HU^{l_q}}\sum_{\tilde{j}_p\in C_{HT^{l_p}}^{p}\tilde{j}_p\neq j_p} CM_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p}e_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p}\end{aligned} \tag{78}$$

5.3. Cost Models Utilized for Implementation of Strategy III. In this case, the total annual utility cost of the multiplant HEN (TUC′) and the total capital cost of all bypasses ($NTCC_2$) can be determined according to eqs 70 and 73, respectively. The other cost models are presented below

$$
\begin{aligned}
\mathrm{ATCC} = {} & \sum_{p=1}^{P} \sum_{(i_p,j_p)\in Y^p} \sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}} \sum_{(\tilde{i}_p,\tilde{j}_p)\in Z^p} \sum_{k\in ST} [CF_{\tilde{i}_p,\tilde{j}_p} \sigma_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}} \\
& + CA_{\tilde{i}_p,\tilde{j}_p} (X_{\tilde{i}_p,\tilde{j}_p,k}^{y_{i_p,j_p}})^{\beta}] \\
& + \sum_{p=1}^{P} \sum_{q'=1}^{P} \sum_{w_p=w_{q'}=1}^{W_p} \sum_{(i_p,n_p)\in CL_p^{w_p}} \sum_{\tilde{i}_p\in H^p} \sum_{\tilde{n}_{q'}\in CU^{w_{q'}}} [CF_{\tilde{i}_p,\tilde{n}_{q'}} \sigma_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} \\
& + CA_{\tilde{i}_p,\tilde{n}_{q'}} (X_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p})^{\beta}] \\
& + \sum_{q=1}^{P} \sum_{p=1}^{P} \sum_{l_p=l_q=1}^{L_p} \sum_{(m_p,j_p)\in HT_p^{l_p}} \sum_{\tilde{m}_q\in HU^{l_q}} \sum_{\tilde{j}_p\in C^p} [CF_{\tilde{m}_q,\tilde{j}_p} \sigma_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} \\
& + CA_{\tilde{m}_q,\tilde{j}_p} (X_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p})^{\beta}]
\end{aligned} \tag{79}
$$

$$
\begin{aligned}
\mathrm{NTCC}_1 = {} & \sum_{p=1}^{P} \sum_{\substack{q'=1 \\ q'\neq p}}^{P} \sum_{i_p\in H^p} \sum_{j_{q'}\in C^{q'}} \sum_{k\in ST} [CF_{i_p,j_{q'}} \xi_{i_p,j_{q'},k} + CA_{i_p,j_{q'}} (A_{i_p,j_{q'}})^{\beta}] \\
& + \sum_{p=1}^{P} \sum_{\substack{q'=1 \\ q'\neq p}}^{P} \sum_{i_p\in H^p} \sum_{n_{q'}\in CU^{q'}} [CF_{i_p,n_{q'}} + CA_{i_p,n_{q'}} (A_{i_p,n_{q'}})^{\beta}] v_{i_p,n_{q'}}^{CU} \\
& + \sum_{q=1}^{P} \sum_{\substack{q=1 \\ q\neq p}}^{P} \sum_{m_q\in HU^q} \sum_{j_p\in C^p} [CF_{m_q,j_p} + CA_{m_q,j_p} (A_{m_q,j_p})^{\beta}] v_{m_q,j_p}^{HU}
\end{aligned} \tag{80}
$$

$$
\begin{aligned}
\mathrm{NTCC}_3 = {} & \sum_{p=1}^{P} \sum_{(i_p,j_p)\in Y^p} \sum_{y_{i_p,j_p}=1}^{N_{i_p,j_p}} \sum_{\substack{(\tilde{i}_p,\tilde{j}_p)\in Z^p \\ \tilde{i}_p\neq i_p \vee \tilde{j}_p\neq j_p}} \sum_{k\in ST} CM_{\tilde{i}_p,\tilde{j}_p}^{i_p,j_p} e_{\tilde{i}_p,\tilde{j}_p,k}^{i_p,j_p,y_{i_p,j_p}} \\
& + \sum_{p=1}^{P} \sum_{q'=1}^{P} \sum_{w_p=w_{q'}=1}^{W_p} \sum_{(i_p,n_p)\in CL_p^{w_p}} \sum_{\substack{\tilde{i}_p\in H^p \\ \tilde{i}_p\neq i_p}} \sum_{\tilde{n}_{q'}\in CU^{w_{q'}}} CM_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} e_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p} \\
& + \sum_{q=1}^{P} \sum_{p=1}^{P} \sum_{l_p=l_q=1}^{L_p} \sum_{(m_p,j_p)\in HT_p^{l_p}} \sum_{\tilde{m}_q\in HU^{l_q}} \sum_{\substack{\tilde{j}_p\in C^p \\ \tilde{j}_p\neq j_p}} CM_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p} e_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p}
\end{aligned} \tag{81}
$$

$$
\begin{aligned}
\mathrm{NTCC}_4 = {} & \sum_{p=1}^{P} \sum_{(i_p,j_p)\in Z^p} \sum_{k\in ST} [CF_{i_p,j_p} + CA_{i_p,j_p} (A_{i_p,j_p,k})^{\beta} u_{i_p,j_p,k}] \\
& + \sum_{p=1}^{P} \sum_{w_p=1}^{W_p} \sum_{(i_p,j_p)\in ZCL_p^{w_p}} [CF_{i_p,n_p} + CA_{i_p,n_p} (A_{i_p,n_p})^{\beta} u_{i_p,n_p}^{CU,w_p}] \\
& + \sum_{p=1}^{P} \sum_{l_p=1}^{L_p} \sum_{(m_p,j_p)\in ZHT_p^{l_p}} [CF_{m_p,i_p} + CA_{m_p,j_p} (A_{m_p,j_p})^{\beta} u_{m_p,j_p}^{HU,l_p}]
\end{aligned} \tag{82}
$$

$$
\begin{aligned}
\mathrm{TUPC} = {} & \sum_{p=1}^{P} \sum_{\substack{q'=1 \\ q'\neq p}}^{P} \sum_{w_p=w_{q'}=1}^{W_p} \sum_{(i_p,n_p)\in CL_p^{w_p}} \sum_{\tilde{i}_p\in H^p} \sum_{\tilde{n}_{q'}\in CU^{w_{q'}}} [CP_{\tilde{i}_p,\tilde{n}_{q'}} e_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p}] \\
& + \sum_{\substack{q=1 \\ q\neq p}}^{P} \sum_{p=1}^{P} \sum_{l_p=l_q=1}^{L_p} \sum_{(m_p,j_p)\in HT_p^{l_p}} \sum_{\tilde{m}_q\in HU^{l_q}} \sum_{\tilde{j}_p\in C^p} [CP_{\tilde{m}_q,\tilde{j}_p} e_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p}]
\end{aligned} \tag{83}
$$

6. CASE STUDIES—MAXIMUM SAVING DESIGNS

As an illustrative example, let us consider three chemical plants (P1, P2, and P3) and their stream data and utility data are given in Tables 1 and 2. Let us further assume that the existing single-plant HENs were synthesized according to the conventional simultaneous optimization strategy[8] and they are presented in Figure 3. The minimum TACs of these HENs were found to be 316,565 USD/yr (P1), 56,294 USD/yr (P2), and 287,769 USD/yr (P3), respectively. The sum of utility costs of all three single-plant HENs, that is, TUC, was determined to be 630,350 USD/yr.

Table 1. Stream Data of the Illustrative Example

plant	stream	T_{IN} (°C)	T_{OUT} (°C)	F (kW/°C)	h (kW/m^2 °C)
P1	H1	150	40	7.0	1.2
	C1	60	140	9.0	1.6
	C2	110	190	8.0	1.0
P2	H1	200	70	5.5	1.5
	C1	30	110	3.5	1.1
	C2	140	190	7.5	1.2
P3	H1	370	150	3.0	1.4
	H2	200	40	5.5	1.1
	H3	110	360	4.5	1.3

Table 2. Utility Data of the Illustrative Example

plant	utility	T_{IN} (°C)	T_{OUT} (°C)	h (kW/m^2 °C)	unit cost (USD/kW yr)
P1	cooling water	25	30	1.2	200
	LP steam	200	200	1.5	375
	MP steam	250	250	1.8	575
	HP steam	300	300	2.3	775
	hot oil	500	475	1.8	900
P2	cooling water	25	30	1.2	250
	LP steam	200	200	1.5	400
	MP steam	250	250	1.8	600
	HP steam	300	300	2.3	800
	hot oil	500	475	1.8	1000
P3	cooling water	25	30	1.2	150
	LP steam	200	200	1.5	350
	MP steam	250	250	1.8	550
	HP steam	300	300	2.3	750
	Hot oil	500	475	1.8	850

The parameters used in both the above and also the subsequent calculations are listed as follows:

- Lower bounds of heat duties (LQ_{i_p,j_p}, $LQ_{i_p,j_{q'}}$, LQ_{i_q,j_p}, LQ_{i_p,n_p}, $LQ_{i_p,n_{q'}}$, LQ_{m_p,j_p}, and LQ_{m_q,j_p}) are all set to be: 30 kW.
- Lower bounds of match flow fractions ($LRH_{i_p,j_p,k}$, $LRH_{i_p,j_{q'},k}$, $LRC_{i_p,j_p,k}$, and $LRC_{i_q,j_p,k}$) are all set to be: 0.1.
- Lower bounds of bypass flow fraction ($LRRC_{j_p,k}$, $LRRCU_{j_p}$, $LRRH_{i_p,k}$, and $LRRHU_{i_p}$) are all set to be: 0.1.
- Lower bounds of heat transfer areas for the augmented units (LX_{i_p,j_p}, $LX_{i_p,n_{q'}}$, and LX_{m_q,j_p}) are all set to be: 1 m^2.
- Based on a life length of 10 years and an arbitrarily chosen yearly interest rate of 5.85%, an annualization factor (af) of 0.1349 is adopted in this study.

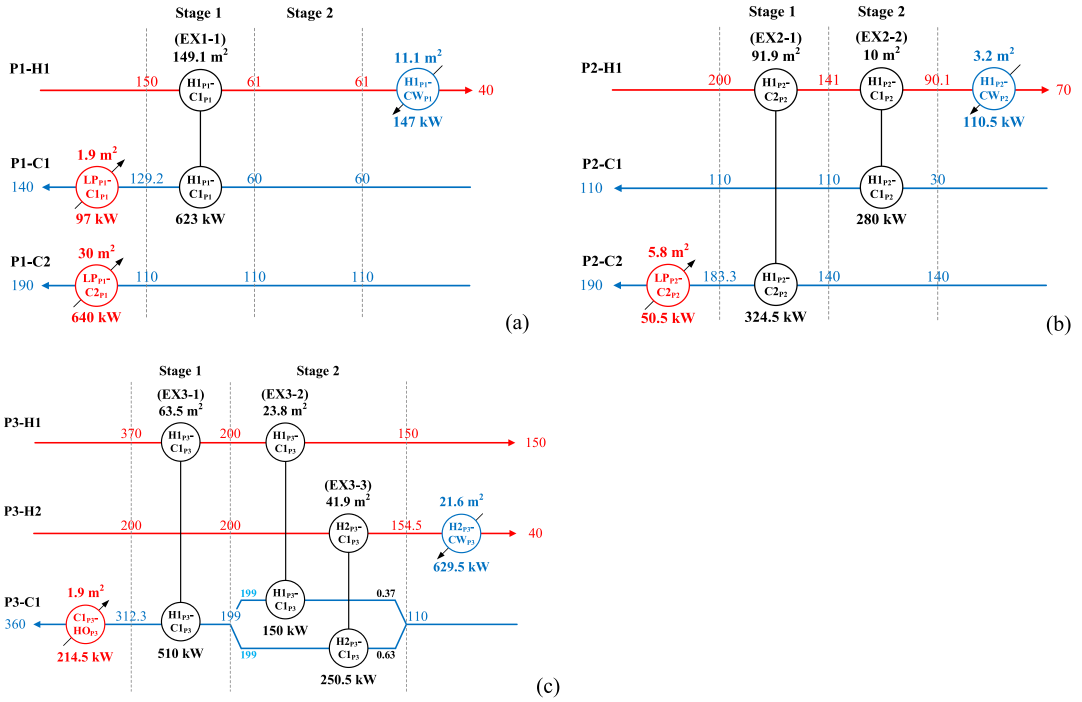

Figure 3. Single-plant HENs of (a) P1, (b) P2, and (c) P3.

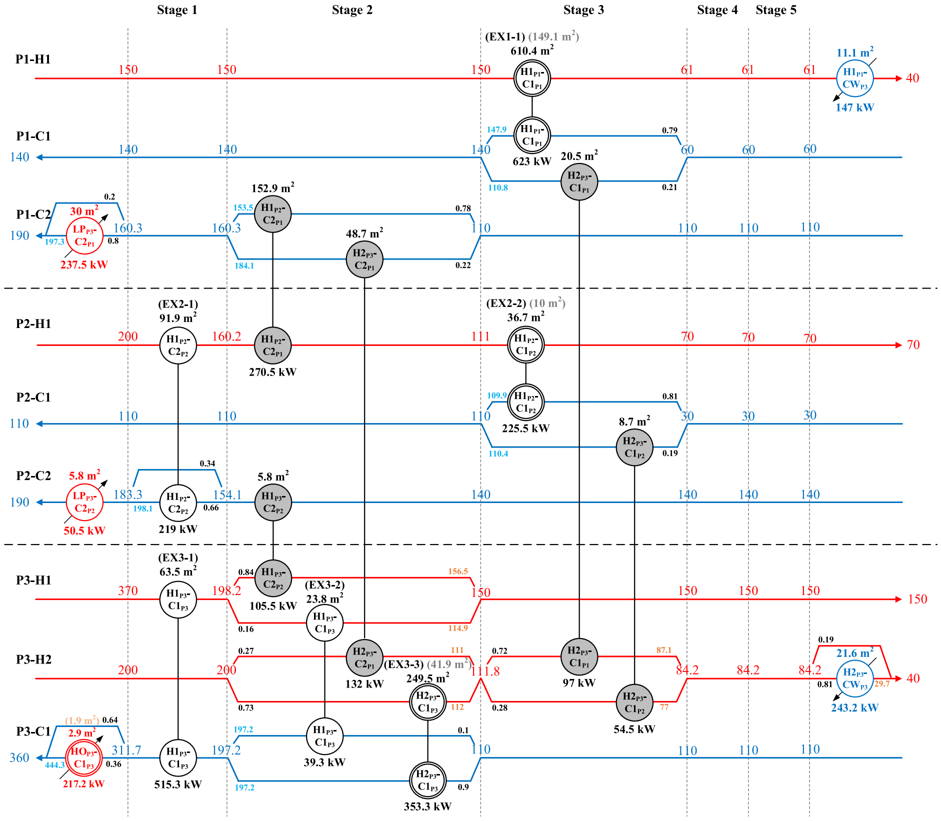

Figure 4. Interplant heat integration scheme obtained by revamping the single-plant HENs of P1, P2, and P3 with Strategy I.

- The exponent of area in the variable cost term (β) is 0.83.
- The variable cost coefficients of the heat exchangers ($CA_{i_pj_p}$, $CA_{i_pj_{q'}}$, and $CA_{i_qj_p}$) and those of coolers ($CA_{i_pn_p}$ and $CA_{i_pn_{q'}}$) are all set to be: 380 $/m$^{1.66}$; the variable cost coefficients of the heaters ($CA_{m_pj_p}$ and $CA_{m_qj_p}$) are all set to be: 700 USD/m$^{1.66}$.
- The fixed costs of interplant heat exchangers, coolers, and heaters ($CF_{i_pj_{q'}}$, $CF_{i_qj_p}$, $CF_{i_pn_{q'}}$ and $CF_{m_qj_p}$) are all set to be: 30,000 USD; The fixed costs of all inner-plant

https://dx.doi.org/10.1021/acs.iecr.0c03829
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units (CF_{i_p,j_p}, CF_{i_p,n_p}, and CF_{m_p,j_p}) are all assumed to be: 10,000 USD.

- The repiping cost of every bypass (BY_{i_p} and BY_{j_p}) is 500 USD.
- The reassignment cost for existing units ($CM_{\tilde{i}_p,\tilde{j}_p}^{i_p,j_p}$, $CM_{\tilde{i}_p,\tilde{n}_{q'}}^{i_p,n_p,w_p}$, and $CM_{\tilde{m}_q,\tilde{j}_p}^{m_p,j_p,l_p}$) are all set to be: 2000 $.
- The fixed costs of pipes for adopting utilities from other plants ($CP_{i_p,n_{q'}}$ and CP_{m_q,j_p}) are all set to be: 5000 $.

Finally, it should be noted that the optimal solutions presented in the sequel were all obtained with solver BARON in GAMS 27.3 on a personal computer (Intel Core i7 6700; 16G).

6.1. Optimal Solution Obtained with Strategy I.

An MINLP model can be constructed according to the formulations described in Part A of Supporting Information, subsections 4.1, and 5.1. The actual numbers of real and integer variables in the GAMS code were 1503 and 466, respectively, while that of constraints was 3167. After solving this model, an optimal revamped design of the three-plant HEN can be generated (see Figure 4). The computation time in this case was around 31,000 s. Notice that every interplant match in Figure 4 is represented with vertically connected circles filled with gray color and each should be housed in a purchased new heat exchanger. All inner-plant matches are represented with vertically connected circles without color and they are housed (at least partially) in the existing heat exchangers. The inner-plant matches that require augmented units are indicated with double circles, while the others are marked with single circles. Table 3 shows the arrangements of new units, which house interplant matches. The net saving of this design, that is, TACS, was determined to be 239,218 USD/yr, while the sum of utility costs of all single-plant HENs, that is, TUC, and the sum of the yearly utility cost and annualized total capital investment of the retrofit design, that is, TUC′ + af(ATCC + NTCC$_1$ + NTCC$_2$ + NTCC$_3$ + NTCC$_4$ + TUPC), were 630,350 USD/yr (a constant parameter) and 391,132 USD/yr, respectively. Finally, by comparing Figures 3 and 4, it can be observed that a large amount of utility saving can be realized via retrofit, that is

Table 3. Assignments of New Units Using Strategy I

interplant matches using new heat exchangers in retrofit design $(i_p, j_{q'})$ or (i_q, j_p)	areas of new heat exchanger (m^2)
$(H1_{P2}, C2_{P1})$	152.9
$(H1_{P3}, C2_{P2})$	5.8
$(H2_{P3}, C1_{P1})$	20.5
$(H2_{P3}, C2_{P1})$	48.7
$(H2_{P3}, C1_{P2})$	8.7

- The hot utilities used on the two cold streams in plant P1 can be reduced significantly, that is, the heat duty on $C1_{P1}$ is decreased from 97 to 0 kW and the heat duty on $C2_{P1}$ from 640 kW to 237.5 kW;
- The cold utility used on hot stream $H1_{P2}$ in plant P2 can be reduced from 110.5 kW to 0 kW;
- The cold utility used on hot stream $H2_{P3}$ in plant P3 can be reduced from 629.5 kW to 243.2 kW.

6.2. Optimal Solution Obtained with Strategy II.

Another MINLP model was constructed according to the formulations described in Part A of Supporting Information, subsections 4.2, and 5.2. The actual numbers of real and integer variables in the GAMS code were 1514 and 558, respectively, while that of constraints was 3252. After solving this model, an optimal revamped design of the three-plant HEN can be generated and this design is presented in Figure 5. The computation time in this case was around 134,000 s. The symbols used in Figure 5 follow exactly the same conventions described in subsection 6.1. Because Strategy II allows assignment of every existing heat exchanger, a cooler, or a heater to any existing match of the same type within the same plant, additional information can be extracted from the optimal solution. Table 4 shows the placement scheme of the existing units in the retrofit design, while Table 5 shows the arrangements of new units. Notice from Table 3 that the two existing heat exchangers used to house two matches $(H1_{P2},C1_{P2})$ and $(H1_{P2},C2_{P2})$, respectively, in the original single-plant design of P2 are switched in the revamp design. Notice also from Table 4 that the match $(H1_{P1},CW_{P1})$ is not housed in its original cooler in the retrofit design and, instead, the hot stream $H1_{P1}$ is matched with the cheapest cooling water from plant P3. Similarly, the matches $(LP_{P1},C2_{P1})$ and $(LP_{P2},C2_{P2})$ are also not housed in their original heaters in the retrofit design and, instead, the cold streams $C2_{P1}$ and $C2_{P2}$ are matched with the cheapest low-pressure steam from plant P3. Furthermore, the existing cooler for original match $(H1_{P2},CW_{P2})$ and existing heater for original match $(LP_{P1},C1_{P1})$ are not needed in the retrofit design. Finally, the net saving of this second design, that is, TACS, was determined to be 239,399 USD/yr, while the sum of utility costs of all single-plant HENs, that is, TUC, and the sum of the yearly utility cost and annualized total capital investment of the retrofit design, that is, TUC′ + af(ATCC + NTCC$_1$ + NTCC$_2$ + NTCC$_3$ + NTCC$_4$ + TUPC), were 630,350 USD/yr (a constant parameter) and 390,951 USD/yr, respectively. Finally, by comparing Figures 4 and 5, it can be observed that

- The HEN structures obtained with Strategies I and II are essentially the same, while the total utility consumption rates in both cases are almost equal;
- Although it is necessary to include the extra capital cost for the aforementioned reassignment of exchanger units in plant P2 so as to implement Strategy II, the total capital cost of augmented units and new interplant units of plant P1 for Strategy I is higher. Because the extra capital cost of the former is lower than that of the latter, Strategy II achieved a slightly larger TAC saving when compared with Strategy I.

6.3. Optimal Solution Obtained with Strategy III.

A third MINLP model was constructed according to the formulations described in Part A of Supporting Information, subsections 4.3, and 5.3. The actual numbers of real and integer variables in the GAMS code were 1532 and 694, respectively, while that of constraints was 3327. After solving this model, an optimal revamped design of the three-plant HEN can be generated according to Strategy III and this design is presented in Figure 6. The computation time in this case was around 165,000 s. The symbols used in Figure 6 follow exactly the same conventions described in subsection 6.1, while the new inner-plant matches housed in the new purchased units are represented with vertically connected circles filled with green color. Because Strategy III allows assignment of every existing heat exchanger, cooler, or heater

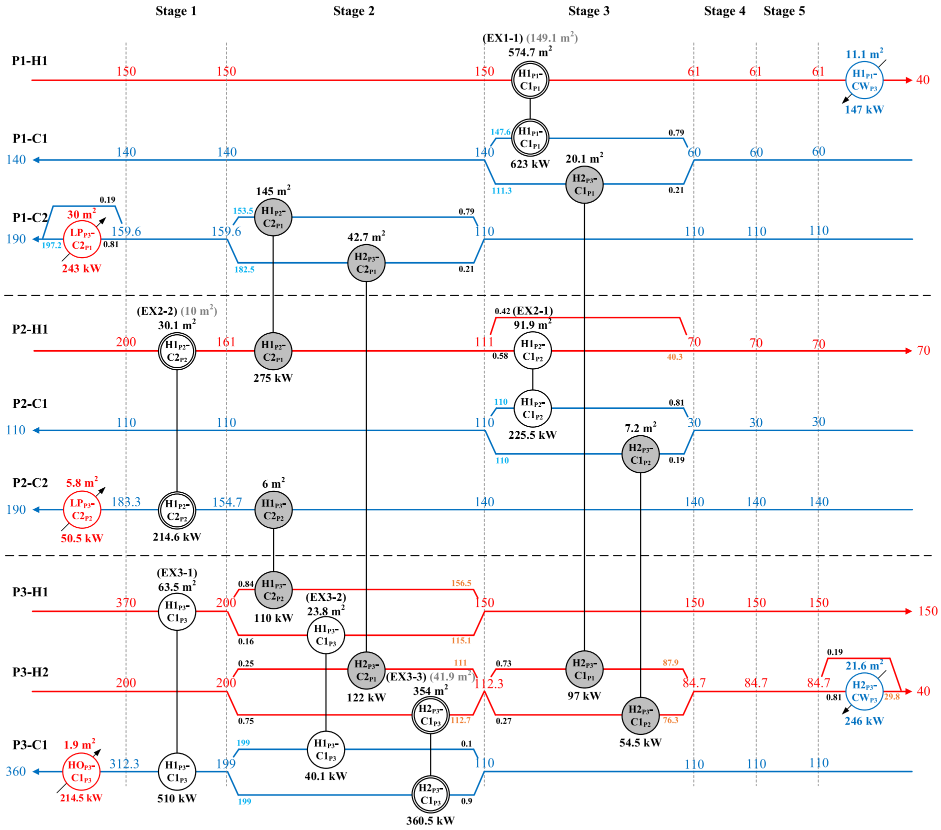

Figure 5. Multiplant HEN design obtained by revamping the single-plant HENs of P1, P2, and P3 with Strategy II.

Table 4. Assignments of Existing Units Using Strategy II

existing matches/exchangers before retrofit (i_p, j_p)	exchanger order $(y_{i_p}, _{j_p})$	assignments of exchanger matches in retrofit design $(\tilde{i}_p, \tilde{j}_p)$	areas of existing heat exchangers (m^2)
$(H1_{P1}, C1_{P1})$	(1)	$(H1_{P1}, C1_{P1})$	149.1
$(H1_{P2}, C1_{P2})$	(1)	$(H1_{P2}, C1_{P2})$	91.9
$(H1_{P2}, C2_{P2})$	(1)	$(H1_{P2}, C1_{P2})$	10
$(H1_{P3}, C1_{P3})$	(1)	$(H1_{P3}, C1_{P3})$	63.5
$(H1_{P3}, C1_{P3})$	(2)	$(H1_{P3}, C1_{P3})$	23.8
$(H1_{P3}, C1_{P3})$	(1)	$(H1_{P3}, C1_{P2})$	41.9
existing matches/coolers before retrofit (i_p, n_p)	cooler type (w_p)	assignments of cooler matches in retrofit design $(\tilde{ip}, \tilde{n}_q)$	areas of existing coolers (m^2)
$(H1_{P1}, CW_{P1})$	(1)	$(H1_{P1}, CW_{P3})$	11.1
$(H1_{P2}, CW_{P2})$	(1)		3.2
$(H1_{P3}, CW_{P3})$	(1)	$(H1_{P3}, CW_{P3})$	21.6
existing matches/heaters before retrofit (m_p, j_p)	heater type (l_p)	assignments of heater matches in retrofit design $(\tilde{m}_q, \tilde{j}_p)$	areas of existing heaters (m^2)
$(L1_{p1}, C1_{P1})$	(1)		1.9
$(L1_{p1}, C2_{P1})$	(1)	$(LP_{P3}, C2_{P1})$	30
$(L1_{p2}, C2_{P2})$	(1)	$(LP_{P3}, C2_{P2})$	5.8
$(HO_{p3}, C1_{P3})$	(2)	$(HO_{P3}, C1_{P3})$	1.9

Table 5. Assignments of New Units Using Strategy II

interplant matches using new heat exchangers in retrofit design $(i_p, j_{q'})$ or (i_q, j_p)	areas of new heat exchanger (m^2)
$(H1_{P2}, C2_{P1})$	145
$(H1_{P3}, C2_{P2})$	6
$(H2_{P3}, C1_{P1})$	20.5
$(H2_{P3}, C2_{P1})$	42.7
$(H2_{P3}, C1_{P2})$	7.2

to any match of the same type within the same plant, additional information can be extracted from the optimal solution. Table 6 shows the assignments of the existing units in the retrofit design, while Table 7 shows the arrangements of new units. Notice from Table 6 that the two existing heat exchangers used to house two matches $(H1_{P2},C1_{P2})$ and $(H1_{P2},C2_{P2})$, respectively, in the original single-plant design of P2 are switched in the revamp design. Notice also from Table 7 that inner-plant match $(H2_{P3},C1_{P3})$ is housed in new purchased heat exchanger. On the other hand, the existing cooler for original match $(H1_{P2},CW_{P2})$ and the existing heater for original match $(LP_{P1},C1_{P1})$ are not needed in the retrofit design, while hot stream $H1_{P1}$ and cold streams $C2_{P1}$ and $C2_{P2}$ are matched with the cheapest utilities from plant P3 (see Table 6). Finally, the net TAC saving of the last design, that is, TACS, was found to be the highest among all three strategies considered so far (i.e., 241,947 USD/yr), while the sum of utility costs of all single-plant HENs, that is, TUC, was 630,350 USD/yr (a constant parameter) and the sum of the yearly utility cost and annualized total capital investment of the retrofit design, that is, TUC′ + af(ATCC + $NTCC_1$ + $NTCC_2$ + $NTCC_3$ + $NTCC_4$ + TUPC), was the lowest at 388,403 USD/yr. Finally, by comparing Figures 5 and 6, it can be observed that.

- The HEN structures obtained with Strategies II and III are similar but not identical, while the total utility consumption rates in both cases are almost equal;
- When compared with the HEN design generated with Strategy II, the design produced with Strategy III needs

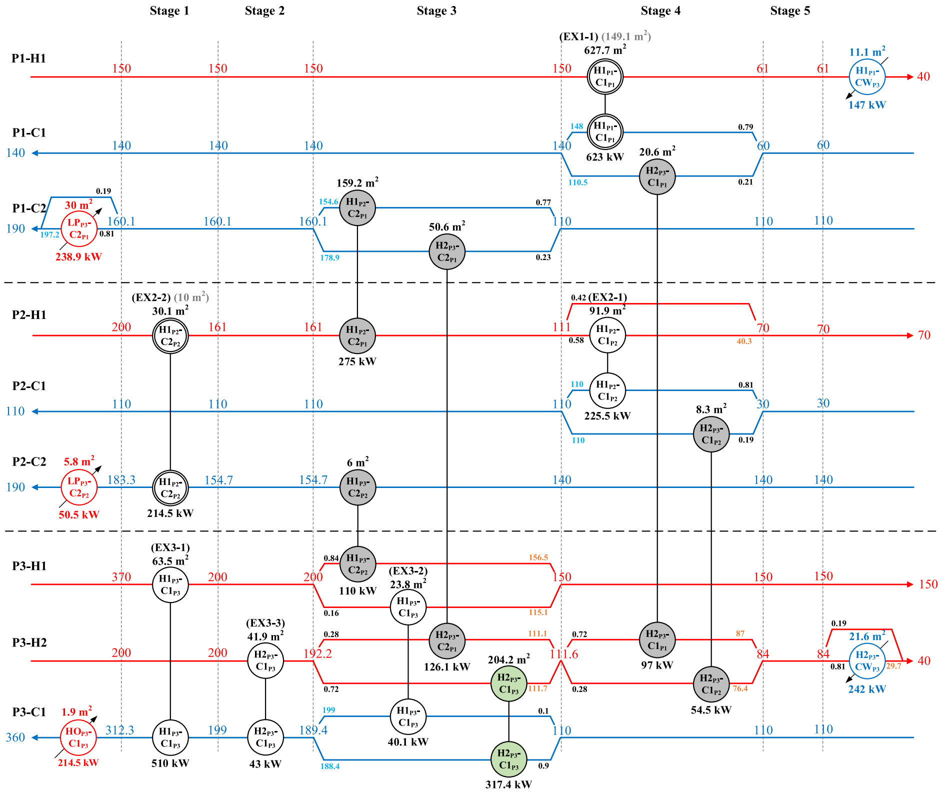

Figure 6. Multiplant HEN design obtained by revamping the single-plant HENs of P1, P2, and P3 with Strategy III.

Table 6. Assignments of Existing Units Using Strategy III

existing matches/exchangers before retrofit (i_p, j_p)	Exchanger order (y_{i_p, j_p})	assignments of exchanger matches in retrofit design $(\tilde{i}_p, \tilde{j}_p)$	areas of existing heat exchangers (m^2)
$(H1_{P1}, C1_{P1})$	(1)	$(H1_{P1}, C1_{P1})$	149.1
$(H1_{P2}, C1_{P2})$	(1)	$(H1_{P2}, C1_{P2})$	91.9
$(H1_{P2}, C1_{P2})$	(1)	$(H1_{P2}, C1_{P2})$	10
$(H1_{P3}, C1_{P3})$	(1)	$(H1_{P3}, C1_{P3})$	63.5
$(H1_{P3}, C1_{P3})$	(2)	$(H1_{P3}, C1_{P3})$	23.8
$(H1_{P3}, C1_{P3})$	(1)	$(H1_{P3}, C1_{P2})$	41.9
existing matches/coolers before retrofit (i_p, n_p)	cooler type (w_p)	assignments of cooler matches in retrofit design $(\tilde{i}_p, \tilde{n}_{q'})$	areas of existing coolers (m^2)
$(H1_{P1}, CW_{P1})$	(1)	$(H1_{P1}, CW_{P3})$	11.1
$(H1_{P2}, CW_{P2})$	(1)		3.2
$(H1_{P3}, CW_{P3})$	(1)	$(H1_{P3}, CW_{P3})$	21.6
existing matches/heaters before retrofit (m_p, j_p)	heater type (l_p)	assignments of heater matches in retrofit design $(\tilde{m}_q, \tilde{j}_p)$	areas of existing heaters (m^2)
$(L1_{p1}, C1_{P1})$	(1)		1.9
$(L1_{p1}, C2_{P1})$	(1)	$(LP_{P3}, C2_{P1})$	30
$(L1_{p2}, C2_{P2})$	(1)	$(LP_{P3}, C2_{P2})$	5.8
$(HO_{p3}, C1_{P3})$	(2)	$(HO_{P3}, C1_{P3})$	1.9

Table 7. Assignments of New Units Using Strategy III

matches using new heat exchangers in retrofit design $(\tilde{i}p, \tilde{j}_p)$ $(\tilde{i}_{p'}, \tilde{j}_{q'})$ or $(\tilde{i}_q, \tilde{j}_p)$	areas of new heat exchanger (m^2)
$(H2_{P3}, C1_{P3})$	204.2
$(H1_{P3}, C2_{P1})$	159.2
$(H1_{P3}, C2_{P2})$	6
$(H2_{P3}, C2_{P1})$	20.6
$(H2_{P3}, C1_{P2})$	8.3

one extra purchased unit for an inner-plant exchanger match in plant P3.

- Although it is necessary to buy a new unit for the above purpose, the capital investment of augmented units for Strategy III is significantly less than that for Strategy II.

7. CORE AND SHAPLEY VALUES

The retrofit design of the above multiplant HEN actually lacks one critical component. In particular, only the total saving of the entire system, that is, TACS, is determined by solving the corresponding MINLP model, while the practical issues of benefit allocation are not addressed at all. In the present work, this allocation problem is viewed as a cooperative game and all players of the game form a so-called "coalition." Typically, the core and Shapley values are used to characterize the reasonable and fair solution(s) for distributing the financial benefit within the coalition. Although extensive discussions on their evaluation procedures have already been published, for example, see Branzei et al.,[17] a brief summary is still given in the sequel for the sake of illustration clarity.

"Core" is the solution set of a co-operative game. Each solution in the set depicts a realizable plan for every member of the coalition to receive a reasonable portion of total cost saving (TACS) after retrofitting. To facilitate illustration, let us use $\Psi = \{1,2,\ldots,n\}$ to represent the set of all players in a game and $\Sigma \subseteq \Psi$ denotes a coalition. Then, all possible coalitions should form the power set of Ψ (denoted as 2^{Ψ}) and a characteristic function $\nu(\cdot)$ can be defined accordingly as the mapping ν: $2^{\Psi} \to R$. The function value $\nu(\Sigma)$, where $\Sigma \in 2^{\Psi}$, is the TACS realized by coalition Σ as a whole. To ensure function consistency, it is also required that $\nu(\emptyset) = 0$. Let us further denote the annual cost saving allocated to plant $i \in \Psi$ in coalition $\Sigma \subseteq \Psi$ as $x_{\Sigma,i}$. Thus, $\nu(\Sigma) = \sum_{i \in \Sigma} x_{\Sigma,i}$ and $\mathbf{x}_{\Sigma} =$

$[x_{\Sigma,1}, x_{\Sigma,2}, ...]$ is referred to as the preimputation vector of coalition Σ. The preimputation vector of grand coalition Ψ, that is, $\mathbf{x}_\Psi$, in the core $C(\nu)$ should possess the following properties.

•Individual rationality. The cost saving allocated to player i in the grand coalition Ψ should be larger than or equal to that achieved by a single player individually, that is

$$x_{\Psi,i} \geq \nu(i), \qquad \forall\ i \in \Psi \tag{84}$$

•Group rationality. The TACS realized by the grand coalition should be entirely distributed to all its members, that is

$$\sum_{i \in \Psi} x_{\Psi,i} = \nu(\Psi) \tag{85}$$

•Coalition rationality. The TACS realized by a subcoalition should not be greater than the sum of cost savings allocated to the members of this subcoalition by the grand coalition, that is

$$\sum_{i \in \Sigma} x_{\Psi,i} \geq \nu(\Sigma), \qquad \forall\ \Sigma \subseteq \Psi \tag{86}$$

• No cross subsidization. The cost saving allocated to player i by coalition Ψ should be smaller than or equal to the marginal contribution of player i to the TACS of coalition Ψ, that is

$$x_{\Psi,i} \leq \nu(\Psi) - \nu(\Psi/i), \qquad \forall\ i \in \Psi \tag{87}$$

Notice that $\nu(\Psi/i)$ above denotes the TACS achieved by the subcoalition of the grand coalition Ψ and this subcoalition is formed by excluding player i from Ψ. In other words, eq 87 is needed because, if otherwise, the members in Ψ/i do not have the incentive to accept player i.

It is clear that the core $C(\nu)$ only represents a feasible region. A one-point solution can be obtained by computing the Shapley values. This allocation approach essentially calls for dividing and distributing the TACS of a coalition according to the contribution level of each participating member. Before evaluating these so-called Shapley values for benefit allocation, it is necessary to calculate the cost savings achieved by all possible subcoalitions $\Sigma \subseteq \Psi$. To enumerate all scenarios exhaustively, let us first consider the $n!$ permutations of the n players in Ψ and collect the corresponding sequences in set $\Pi(\Psi)$. Let us further express an element in $\Pi(\Psi)$ as π_σ (where, $\sigma = 1,2,...,n!$), that is, $\Pi(\Psi) = \{\pi_1, \pi_2, ..., \pi_{n!}\}$, while a particular sequence σ' in $\Pi(\Psi)$ may be written explicitly as $\pi_{\sigma'} = (\pi_{\sigma'}(1), \pi_{\sigma'}(2), ..., \pi_{\sigma'}(n))$. A sequence of marginal contributions of the TACS (denoted as $\mathbf{m}^\sigma$) can then be computed for every sequence $\boldsymbol{\pi}_\sigma$ in $\Pi(\Psi)$, that is

$$\mathbf{m}^\sigma = \{m^\sigma_{\pi_\sigma(1)}, m^\sigma_{\pi_\sigma(2)}, ..., m^\sigma_{\pi_\sigma(k)}, ..., m^\sigma_{\pi_\sigma(n)}\} \tag{88}$$

where

$$m^\sigma_{\pi_\sigma(1)} = \nu(\pi_\sigma(1)) - \nu(\emptyset) \tag{89}$$

$$m^\sigma_{\pi_\sigma(k)} = \nu(\pi_\sigma(1), ..., \pi_\sigma(k)) - \nu(\pi_\sigma(1), ..., \pi_\sigma(k-1)) \tag{90}$$

and $k = 2,3,...,n$. It should be noted that the precedence order of the elements in sequence $\mathbf{m}^\sigma$ corresponds to that in sequence π_σ. These elements can be rearranged according the original order in Ψ and then placed in another column vector $\mathbf{o}^\sigma$. After obtaining $\mathbf{o}^\sigma$ that stores the rearranged marginal contributions for every sequence π_σ in $\Pi(\Psi)$, one can then compute the corresponding arithmetic averages and store them in the Shapley-value vector φ_N as follows

$$\boldsymbol{\varphi}_\Psi = \frac{1}{n!} \sum_{\pi_\sigma \in \pi(\Psi)} \mathbf{o}^\sigma \tag{91}$$

where $\boldsymbol{\varphi}_\Psi = [\varphi_{\Psi,1} \varphi_{\Psi,2} ... \varphi_{\Psi,n}]^T$ and $\varphi_{\Psi,i}$ (where, $i = 1,2,...,n$) denotes the average benefit (cost saving) allocated to player i by coalition Ψ. It should also be noted that the above notation on the Shapley values can be extended to any subset of the grand coalition, that is, $\Sigma \subseteq \Psi$. If the players in Σ form a coalition, then the Shapley value of player i ($\forall i \in \Sigma$) can be written as $\varphi_{\Sigma,i}$.

To summarize, the required procedure for computing the Shapley values in the present application is outlined below:

1. Determine the TACSs of coalitions formed by all possible combinations of plants according to the proposed MINLP models;
2. Determine the marginal benefits of each plant according to the results obtained in step 1 and eqs 89 and 90 for all possible precedence orders of this plant joining the coalition.
3. Determine the Shapley value of each plant by taking the arithmetic average of the marginal benefits obtained in step 2 according to eq 91.

8. CASE STUDIES—BENEFIT ALLOCATION

To illustrate the aforementioned computation procedure, let us revisit the retrofit problem discussed in Section 6. As mentioned before in Section 7, the function value $\nu(\Sigma)$, where $\Sigma \in 2^\Psi$ and $\Sigma \subseteq \Psi$, should be viewed as the TACS realized by the corresponding coalition. In the present case, $\Psi = \{P1, P2, P3\}$ and let us further define the following subsets of Ψ:

- ${\Sigma_1}^1 = \{P1\}$, ${\Sigma_2}^1 = \{P2\}$, ${\Sigma_3}^1 = \{P3\}$;
- ${\Sigma_1}^2 = \{P1, P2\}$, ${\Sigma_2}^2 = \{P2, P3\}$, ${\Sigma_3}^2 = \{P3, P1\}$.

Thus, the Shapley-value vector can be written as

$$\phi_\Psi = \begin{bmatrix} \varphi_{\Psi,P1} \\ \varphi_{\Psi,P2} \\ \varphi_{\Psi,P3} \end{bmatrix} = \begin{bmatrix} \dfrac{2\nu(\Psi) + \nu({\Sigma_3}^2) + \nu({\Sigma_1}^2) + 2\nu({\Sigma_1}^1) - 2\nu({\Sigma_2}^2) - \nu({\Sigma_2}^1) - \nu({\Sigma_3}^1)}{6} \\ \dfrac{2\nu(\Psi) + \nu({\Sigma_1}^2) + \nu({\Sigma_2}^2) + 2\nu({\Sigma_2}^1) - 2\nu({\Sigma_3}^2) - \nu({\Sigma_3}^1) - \nu({\Sigma_1}^1)}{6} \\ \dfrac{2\nu(\Psi) + \nu({\Sigma_2}^2) + \nu({\Sigma_3}^2) + 2\nu({\Sigma_3}^1) - 2\nu({\Sigma_1}^2) - \nu({\Sigma_1}^1) - \nu({\Sigma_2}^1)}{6} \end{bmatrix} \tag{92}$$

The allocation schemes of cost savings achieved with different retrofit strategies can all be devised according to eq 92.

8.1. Allocation of Cost Saving Achieved with Strategy I. By repeated solving the MINLP models described in Part A of the Supporting Information, subsections 4.1, and 5.1 for coalitions Σ_1^2, Σ_2^2, Σ_3^2, and Ψ, one can obtain the following function values.

- $\nu(\varnothing) = \nu(\Sigma_1^1) = \nu(\Sigma_2^1) = \nu(\Sigma_3^1) = 0$;
- $\nu(\Sigma_1^2) = 106{,}801, \nu(\Sigma_2^2) = 28{,}713, \nu(\Sigma_3^2) = 162{,}366$;
- $\nu(\Psi) = 239{,}218$.

Therefore, the cost savings allocated to P1, P2, and P3 can be computed according to eq 92, that is, $\varphi_{\Psi,\mathrm{P1}} = 115{,}029$ USD/yr (48.1%), $\varphi_{\Psi,\mathrm{P2}} = 48{,}203$ USD/yr (20.1%), and $\varphi_{\Psi,\mathrm{P3}} = 75{,}985$ USD/yr (31.8%).

8.2. Allocation of Cost Saving Achieved with Strategy II. By repeated solving the MINLP models described in Part A of the Supporting Information, subsections 4.2, and 5.2 for coalitions Σ_1^2, Σ_2^2, Σ_3^2, and Ψ, one can obtain the following function values.

- $\nu(\varnothing) = \nu(\Sigma_1^1) = \nu(\Sigma_2^1) = \nu(\Sigma_3^1) = 0$;
- $\nu(\Sigma_1^2) = 108{,}904$, $\nu(\Sigma_2^2) = 28{,}713$, $\nu(\Sigma_3^2) = 162{,}366$;
- $\nu(\Psi) = 239{,}399$.

Therefore, the cost savings allocated to P1, P2, and P3 can be computed according to eq 92, that is, $\varphi_{\Psi,\mathrm{P1}} = 115{,}440$ USD/yr (48.2%), $\varphi_{\Psi,\mathrm{P2}} = 48{,}614$ USD/yr (20.3%), and $\varphi_{\Psi,\mathrm{P3}} = 75{,}345$ USD/yr (31.5%).

8.3. Allocation of Cost Saving Achieved with Strategy III. By repeated solving the MINLP models described in Part A of the Supporting Information, subsections 4.3 and 5.3 for coalitions Σ_1^2, Σ_2^2, Σ_3^2, and Ψ, one can obtain the following function values.

- $\nu(\varnothing) = \nu(\Sigma_1^1) = \nu(\Sigma_2^1) = \nu(\Sigma_3^1) = 0$;
- $\nu(\Sigma_1^2) = 108{,}904$, $\nu(\Sigma_2^2) = 28{,}713$, $\nu(\Sigma_3^2) = 168{,}878$;
- $\nu(\Psi) = 241{,}947$.

Therefore, the cost savings allocated to P1, P2, and P3 can be computed according to eq 92, that is, $\varphi_{\Psi,\mathrm{P1}} = 117{,}375$ USD/yr (48.5%), $\varphi_{\Psi,\mathrm{P2}} = 47{,}293$ USD/yr (19.6%), and $\varphi_{\Psi,\mathrm{P3}} = 77{,}279$ USD/yr (31.9%).

8.4. Implications of Allocation Results. It can be observed from the aforementioned Shapley values that, although the percentages of cost savings allocated to P1 and P3 rise as the model restriction gradually relaxes from Strategy I to III, the share of P2 decreases from Strategy II to III, which means that the average marginal contribution of P2 declines from Strategy II to III. On the other hand, despite the fact that the cost savings achieved by the three-plant HENs for $\nu(\Psi)$ increase in the corresponding three cases, $\Sigma_2^2 = \{\mathrm{P2, P3}\}$ still remains unaffected by the different revamp strategies under consideration. Thus, the above observations seem to suggest that the contribution levels to overall cost saving of the three-plant heat integration scheme can be ranked as (1) P1, (2) P3, and (3) P2 in the present example.

9. CONCLUSIONS

A comprehensive design procedure is proposed in this paper to revamp the multiplant HENs for lowering the overall utility consumption level and to divide and allocate the resulting benefit (cost saving) fairly to all members of the interplant heat integration scheme. Three retrofit strategies are devised to satisfy practical requirements, such as safety issues, space limitations, and/or operability problems, etc. The corresponding MINLP models can be constructed by augmenting the superstructure-based formulation presented in Part A of the Supporting Information with different versions of additional constraints imposed on the new and original matches, on repiping and reusing of the existing units in the multiplant HEN, and on placement of purchased heat exchangers, coolers, and heaters. As expected, it can be observed from the illustrative example that a greater financial gain can always be realized with a less-constrained MINLP model. The allocation plan of overall cost saving is drawn up according to the well-established Shapley values in this study. From the allocation results in the same example, the contribution levels of the players in the corresponding co-operative game can also be easily identified. In summary, the proposed revamp design is indeed cost effective and the associated allocation plan is fair enough if factors other than marginal benefits, for example, risk of coalition collapse,[1] are not important. Therefore, under this condition, the proposed allocation plan should be acceptable to all parties participating in the multiplant HEN retrofit project. Furthermore, the proposed MINLP models and the corresponding calculation procedure for stipulating the benefit allocation plans should be applicable to any practical retrofit design for total-site heat integration.

■ ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acs.iecr.0c03829.

Superstructure-based model constraints and summary of the revamp strategies (PDF)

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Notes

The authors declare no competing financial interest.

■ ACKNOWLEDGMENTS

This work is supported by the Ministry of Science and Technology of the ROC government under grant MOST 109-2221-E-006-085.

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